மனோன்மணியம் சுந்தரனார் பல்கலைக்கழகம்

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DIRECTORATE OF DISTANCE AND CONTINUING EDUCATION

B.Sc. MATHEMATICS

MATHEMATICS FOR COMPETITIVE EXAMINATION III

Sub. Code: JNMA31 **(For Private Circulation only)**

MATHEMATICS FOR COMPETITIVE EXAMINATION III - SYLLABUS

Unit I.

Square root and cube root.

UNIT II.

Trains.

UNIT III.

Problems on age.

UNIT IV.

Area.

UNIT V.

Volume & Surface area.

Recommended Text: R.S. Agarwal -Objective Arithmetic, Published byS. Chand& Co, Ltd., Edition, 2018.

Contents

UNIT – 1

SQUARE ROOTS AND CUBE ROOTS

IMPORTANT FACTS AND FORMULAE:

1. ***Square Root:*** If $x^2 = y$, we say that the square root of y is x and we write, $\sqrt{y} = x$. For example. $\sqrt{4} = 2, \sqrt{9} = 3, \sqrt{196} = 14$.
1. **Cube Root:** The cube root of a given number x is the number whose cube is x. We denote the cube root of x by $\sqrt[3]{x}$.

Note:

1. $\sqrt{xy} = \sqrt{x} \times \sqrt{y}$.

2. $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}} = \frac{\sqrt{x}}{\sqrt{y}} \times \frac{\sqrt{y}}{\sqrt{y}} = \frac{\sqrt{xy}}{y}$.

SOLVED PROBLEMS.

Problem 1. *Evaluate* $\sqrt{6084}$ *by factorization method.*

Solution.

2	6084
2	3042
3	1521
3	507
13	169
	13

Method: Express the given number as the product of prime factors. Now, take the product of these prime factors choosing one out of every pair of the same primes. This product gives the square root of the given number.

Thus, resolving 6084 into prime factors, we get:

$6084 = 2^2 \times 3^2 \times 13^2$

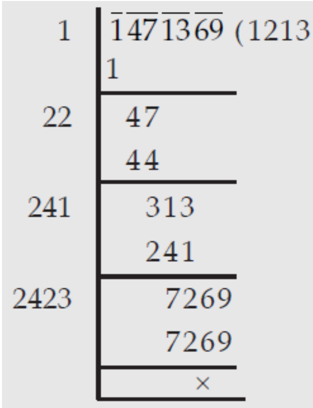
$\therefore \sqrt{6084} = (2 \times 3 \times 13) = 78.$

Problem. 2. *Find the square root of 1471369.*

Solution.

Method: In the given number, mark off the digits in pairs starting from the unit's digit. Each pair and the remaining one digit is called a period.

Now, $1^2 = 1$. On subtracting, we get 0 as remainder.



Now, bring down the next period i.e., 47.

Now, trial divisor is 1 × 2 = 2 and trial dividend is 47.

So, we take 22 as divisor and put 2 as quotient.

The remainder is 3.

Next, we bring down the next period which is 13.

Now, trial divisor is 12 × 2 = 24 and trial dividend is 313. So, we take 241 as dividend and 1 as quotient.

The remainder is 72.

Bring down the next period i.e., 69.

Now, the trial divisor is 121 × 2 = 242 and the trial dividend is 7269. So, we take 3 as quotient and 2423 as divisor. The remainder is then zero.

Hence, $\sqrt{1471369} = 1213$.

Problem 3. *Evaluate:* $\sqrt{248 + \sqrt{52 + \sqrt{144}}}$.

Solution.

Given expression $= \sqrt{248 + \sqrt{52 + 12}} = \sqrt{248 + \sqrt{64}} = \sqrt{248 + 8} = \sqrt{256} = 16$.

Problem 4. *Simplify:* $\frac{112}{\sqrt{576}} \times \frac{\sqrt{576}}{12} \times \frac{\sqrt{256}}{8}$.

Solution.

Given expression = : $\frac{112}{14} \times \frac{24}{12} \times \frac{16}{8}$= 8 × 2 × 2 = 32.

Problem 5. *If* $a * b * c = \frac{\sqrt{(a+2)(b+3)}}{c+1}$, *then find the value of 6 * 15 * 3.*

Solution. $6 * 15 * 3 = \frac{\sqrt{(6+2)(15+3)}}{3+1} = \frac{\sqrt{(8)(18)}}{4} = \frac{\sqrt{144}}{4} = \frac{12}{4} = 3.$

Problem 6. *Find the value of* $\sqrt{1\frac{9}{16}}$.

Solution.

$\sqrt{1\frac{9}{16}} = \sqrt{\frac{25}{16}} = \frac{5}{4} = 1\frac{1}{4}.$

Problem 7. *What is the square root of 0.0009?*

Solution.

$\sqrt{0.0009} = \sqrt{\frac{9}{10000}} = \frac{\sqrt{9}}{\sqrt{10000}} = \frac{3}{100} = 0.03.$

Problem 8. *Evaluate* $\sqrt{175.2976}$.

Solution.

Method: We make even number of decimal places by affixing a zero, if necessary. Now, we mark off periods and extract the square root as shown.

$\therefore \sqrt{175.2976} = 13.24.$

Problem 9. *What will come in place of question mark in each of the following questions*?

(i) $\sqrt{\frac{32.4}{?}}=2.$

(ii) $\sqrt{86.49} + \sqrt{5 + (?)}^2 = 12.3.$

Solution.

(i) $\sqrt{\frac{32.4}{x}} = 2$

Then, $9.3 + \sqrt{5 + x^2} = 12.3$

$\Leftrightarrow \sqrt{5 + x^2} = 12.3 - 9.3 = 3$

$\Leftrightarrow 5 + x^2 = 9$

$\Leftrightarrow x^2 = 9 - 5 = 4$

$\Leftrightarrow x = \sqrt{4} = 2$

Problem 10. *Find the value of* $\sqrt{\frac{0.289}{0.00121}}$.

Solution.

$\sqrt{\frac{0.289}{0.00121}} = \sqrt{\frac{28900}{121}} = \frac{170}{11}.$

Problem 11. *If* $\sqrt{841} = 29$*, then find the value of* $\sqrt{841} + \sqrt{8.41} + \sqrt{0.0841} + \sqrt{0.000841}$.

Solution.

Given expression $= \sqrt{841} + \sqrt{\frac{841}{10^2}} + \sqrt{\frac{841}{10^4}} + \sqrt{\frac{841}{10^6}}$

$= \sqrt{841} + \frac{\sqrt{841}}{10} + \frac{\sqrt{841}}{10^2} + \frac{\sqrt{841}}{10^3}$

$= 29 + \frac{29}{10} + \frac{29}{100} + \frac{29}{1000}$

$= 29 + 2.9 + 0.29 + 0.029$

$= 32.219.$

Problem 12. *If* $\sqrt{1 + \frac{x}{144}} = \frac{13}{12}$*, then find the value of x.*

Solution.

$\sqrt{1 + \frac{x}{144}} = \frac{13}{12}$

$\Rightarrow \left(1+\frac{x}{144}\right) = \left(\frac{13}{12}\right)^2 = \frac{169}{144}$

$\Rightarrow \frac{x}{144} = \frac{169}{144} - 1$

$\Rightarrow \frac{x}{144} = \frac{25}{144}$

$\Rightarrow x = 5.$

Problem 13. *Simplify:* $\frac{1}{\sqrt{100}-\sqrt{99}} - \frac{1}{\sqrt{99}-\sqrt{98}} + \frac{1}{\sqrt{98}-\sqrt{97}} - \frac{1}{\sqrt{97}-\sqrt{96}} + \cdots + \frac{1}{\sqrt{2}-\sqrt{1}}.$

Solution.

Given expression

$= \frac{1}{\sqrt{100}-\sqrt{99}} \times \frac{\sqrt{100}+\sqrt{99}}{\sqrt{100}+\sqrt{99}} - \frac{1}{\sqrt{99}-\sqrt{98}} \times \frac{\sqrt{99}+\sqrt{98}}{\sqrt{99}+\sqrt{98}} + \frac{1}{\sqrt{98}-\sqrt{97}} \times \frac{\sqrt{98}+\sqrt{97}}{\sqrt{98}+\sqrt{97}} - \frac{1}{\sqrt{97}-\sqrt{96}} \times \frac{\sqrt{97}+\sqrt{96}}{\sqrt{97}+\sqrt{96}} +$

$\cdots + \frac{1}{\sqrt{2}-\sqrt{1}} \times \frac{\sqrt{2}+\sqrt{1}}{\sqrt{2}+\sqrt{1}}.$

$= \frac{\sqrt{100}+\sqrt{99}}{100-99} - \frac{\sqrt{99}+\sqrt{98}}{99-98} + \frac{\sqrt{98}+\sqrt{97}}{98-97} - \frac{\sqrt{97}+\sqrt{96}}{97-96} + \cdots + \frac{\sqrt{2}+\sqrt{1}}{2-1}$

$= \left(\sqrt{100}+\sqrt{99}\right) - \left(\sqrt{99}+\sqrt{98}\right) + \left(\sqrt{98}+\sqrt{97}\right) - \cdots + \left(\sqrt{2}+\sqrt{1}\right)$

$= \sqrt{100} + \sqrt{1}$

$= 10 + 1.$

Problem 14. $Find\ the\ sum$: $3 + \frac{1}{\sqrt{3}} + \frac{1}{3+\sqrt{3}} - \frac{1}{3-\sqrt{3}}.$

Solution.

$$3 + \frac{1}{\sqrt{3}} + \frac{1}{3+\sqrt{3}} - \frac{1}{3-\sqrt{3}} = 3 + \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} + \frac{1}{3+\sqrt{3}} \times \frac{3-\sqrt{3}}{3-\sqrt{3}} - \frac{1}{3-\sqrt{3}} \times \frac{3+\sqrt{3}}{3+\sqrt{3}}$$

$$= 3 + \frac{\sqrt{3}}{3} + \frac{3-\sqrt{3}}{9-3} - \frac{3+\sqrt{3}}{9-3}$$

$$= 3 + \frac{\sqrt{3}}{3} + \frac{3-\sqrt{3}}{6} - \frac{3+\sqrt{3}}{6}$$

$$= \frac{18 + 2\sqrt{3} + 3 - \sqrt{3} - 3 - \sqrt{3}}{6}$$

$$= \frac{18}{3} = 3$$

Problem 15. *Find the value of 3 upto three places of decimal.*

Solution.

1	$\overline{3}.\overline{00}\,\overline{00}\,\overline{00}$ (1.732
	1
27	200
	189
343	1100
	1029
3462	7100
	6924

$\therefore \sqrt{3} = 1.732$

Problem 16.

If $\sqrt{3} = 1.732$, find the value of $\sqrt{192} - \frac{1}{2}\sqrt{48} - \sqrt{75}$ correct to 3 places of decimal.

Solution.

$$\sqrt{192} - \tfrac{1}{2}\sqrt{48} - \sqrt{75} = \sqrt{64 \times 3} - \tfrac{1}{2}\sqrt{16 \times 3} - \sqrt{25 \times 3}$$

$$= 8\sqrt{3} - \frac{1}{2} \times 4\sqrt{3} - 5\sqrt{3}$$

$$= 3\sqrt{3} - 2\sqrt{3} = \sqrt{3}$$

Problem 17.

If $\sqrt{0.05 \times 0.5 \times a} = 0.5 \times 0.05 \times \sqrt{b}$ *, then find the value of* $\frac{a}{b}$.

Solution.

Clearly, we have:

$$\frac{\sqrt{a}}{\sqrt{b}} = \frac{0.5 \times 0.05}{\sqrt{0.5 \times 0.05}}$$

$$\Rightarrow \frac{a}{b} = \frac{0.5 \times 0.05 \times 0.05}{0.5 \times 0.05}$$

$$= 0.5 \times 0.05$$

$$= 0.025$$

Problem 18. *Evaluate :* $\sqrt{\frac{9.5\times.0085\times18.9}{.0017\times1.9\times0.021}}$

Solution. Given exp = $\sqrt{\frac{9.5\times.0085\times18.9}{.0017\times1.9\times0.021}}$

Now, since the sum of decimal places in the numerator and denominator under the radical sign is the same, we remove the decimal.

∴Given exp.= $\sqrt{\frac{95\times85\times18900}{17\times19\times21}}$

$= \sqrt{5\times5\times900}$

$= 5\times30$

$= 150$

Problem 19. *Simplify:*$\sqrt{(12.1)^2-(8.1)^2\div(0.25)(19.95)}$

Solution. Given exp. = $\sqrt{\frac{(12.1+8.1)(12.1-8.1)}{(0.25)(0.25+19.95)}}$

$= \sqrt{\frac{20.2\times4}{0.25\times20.2}}$

$= \sqrt{\frac{4}{0.25}}$

$= \sqrt{\frac{400}{25}}$

$= \sqrt{16}$

$= 4$

Problem 20. *If* $x = 1+\sqrt{2}$ *and* $= 1-\sqrt{2}$ *, find the value of* $(x^2 + y^2)$.

Solution.

$x^2 + y^2 = \left(1+\sqrt{2}\right)^2 + \left(1-1+\sqrt{2}\right)^2$

$= 2\,[(1)^2 + (2)^2]$

$= 2 \times 3 = 6.$

Problem 21. *Find the square root of 0.1*

Solution.

$\sqrt{0.1} = \sqrt{\frac{1}{9}} = \frac{1}{3} = \cdots. = 0.333\ldots = 0.3$

Problem 22.

Find the least square number which is exactly divisible by 10, 12, 15 and 18.

Solution.

L.C.M. of 10, 12, 15, 18 = 180.

Now, 180 = 2 × 2 × 3 × 3 × 5 = 22 × 32 × 5.

To make it a perfect square, it must be multiplied by 5.

∴ Required number = $(2^2 \times 3^2 \times 5^2) = 900.$

Problem 23. *Find the greatest number of five digits which is a perfect square.*

Solution. Greatest number of 5 digits is 99999.

3	$\overline{9}\,\overline{99}\,\overline{99}$ (316
	9
61	99
	61
626	3899
	3756
	143

∴ Required number = $(99999 - 143) = 99856.$

Problem 24. *Find the smallest number that must be added to 1780 to make it a perfect square.*

Solution.

```
4 | 1780 (42
  | 16
  |------
82|  180
  |  164
  |------
  |   16
  |------
```

∴Number to be added = $(43)^2 - 1780 = 1849 - 1780 = 69$

Problem 25. *If* $\sqrt{2} = 1.4142$*, find the value of* $\frac{\sqrt{2}}{(2+\sqrt{2})}$

Solution.

$\frac{\sqrt{2}}{(2+\sqrt{2})} = \frac{\sqrt{2}}{(2+\sqrt{2})} \times \frac{(2-\sqrt{2})}{(2-\sqrt{2})}$

$= \frac{(2\sqrt{2}-1)}{(4-2)}$

$= \frac{(2\sqrt{2}-1)}{2}$

$= \sqrt{2} - 1$

$= (1.4142 - 1)$

$= 0.4142.$

Problem 26.

Find the value of: $\sqrt{6 + \sqrt{6 + \sqrt{6} + \cdots.}}$

Solution.

Let $\sqrt{6 + \sqrt{6 + \sqrt{6} + \cdots.}} = x$

Then, $\sqrt{6 + x} = x$

$\Leftrightarrow 6 + x = x^2$

$\Leftrightarrow x^2 - x - 6 = 0$

$\Leftrightarrow x^2 - 3x + 2x - 6 = 0$

$\Leftrightarrow x (x - 3) + 2 (x - 3) = 0$

$\Leftrightarrow (x - 3) (x + 2) = 0$

$\Leftrightarrow x = 3.$

Hence, $\sqrt{6+\sqrt{6+\sqrt{6}+\cdots.}} = 3.$

Problem 27.

By what least number 4320 be multiplied to obtain a number which is a perfect cube?

Solution.

Clearly, $4320 = 2^3 \times 3^3 \times 2^2 \times 5.$

To make it a perfect cube, it must be multiplied by 2×5^2

i.e., 50.

EXERCISE

1. $\sqrt{53824} = ?$
2. Find the square root of 41209.
3. Find the square root of 123454321.
4. Find the number of digits in the square root of 625685746009.
5. $\sqrt{\sqrt{17956} + \sqrt{24025}} = ?$
6. $\sqrt{\sqrt{44944} + \sqrt{52441}} = ?$
7. One-fourth of the sum of prime numbers, greater than 4 but less than 16, find the square.
8. Evaluate $\sqrt{41 - \sqrt{21+\sqrt{19-\sqrt{9}}}}.$
9. $\left(\sqrt{\frac{225}{729}} - \sqrt{\frac{25}{729}}\right) \div \sqrt{\frac{16}{81}} = ?$
10. $\sqrt{?} \times \sqrt{484} = 1034$

11. In the equation $\frac{4050}{\sqrt{450}}, x\ =$ find the value of *x*.

12. $\left[\left(\sqrt{81}\right)^2\right]^{\mathbf{2}} = (?\,)^{\mathbf{2}}$

13. How many two-digit numbers satisfy this property: The last digit (unit's digit) of the square of the two-digit number is 8?

14. While solving a mathematical problem, Samidha squared a number and then subtracted 25 from it rather than the required i.e., first subtracting 25 from the number and then squaring it. But she got the right answer. What was the given number?

15. What is the square root of 0.16?

16. If $0.13\ \div\ p^2\ =\ 13$, then fins the value of *p*.

17. If $\sqrt{6}\ =\ 2.449$, then find the value of $\frac{3\sqrt{2}}{2\sqrt{3}}$.

18. Find the smallest number to be added to 680621 to make the sum a perfect square.

19. Find the smallest positive integer *n*, for which 864*n* is a perfect cube.

20. If $a = \frac{\sqrt{3}}{2}$, then $\sqrt{1+a}$+$\sqrt{1-a}$ =?

ANSWERS.

(1). 232 (**2).** 203 (**3).** 11111 (**4).** 6 (**5).** 256 (**6).** 441 (**7).** 3

(8)**.** 6 (9)**.** $\frac{5}{16}$ (**10).** 2209 (**11).** 81 (12)**.** 4096 (13)**.** 3 (14). 13

(15)**.** 7 (16)**.** 0.04 (17)**.** 100 (18)**.** 0.6122 (19)**.** 5 (20)**.** 4

UNIT – 2

PROBLEMS ON TRAINS

IMPORTANT FACTS AND FORMULAE.

I. $a\ km/hr = \left(a \times \frac{5}{18}\right)$ ms.

II. $a\ m/s = \left(a \times \frac{18}{5}\right)$ km/hr.

III. Time taken by a train of length *l* metres to pass a pole or a standing man or a signal post is equal to the time taken by the train to cover *l* metres.

IV. Time taken by a train of length *l* metres to pass a stationary object of length *b* metres is the time taken by the train to cover (*l* + *b*) metres.

V. Suppose two trains or two bodies are moving in the same direction at *u* m/s and *v* m/s, where *u* > *v*, then their relative speed = (*u* – *v*) m/s.

VI. Suppose two trains or two bodies are moving in opposite directions at *u* m / s and *v* m / s, then their relative speed = (*u* + *v*) m / s.

VII. If two trains of length *a* metres and *b* metres are moving in opposite directions at *u* m / s and *v* m / s, then time taken by the trains to cross each other $= \frac{(a+b)}{(u+v)}$ sec.

VIII. If two trains of length *a* metres and *b* metres are moving in the same direction at *u* m / s and *v* m / s, then the time taken by the faster train to cross the slower train $= \frac{(a+b)}{(u-v)}$ sec.

IX. If two trains (or bodies) start at the same time from points *A* and B towards each other and after crossing they take *a* and *b* sec in reaching B and A respectively, then

(*A's speed*) : (*B's speed*) = (*b* : *a*).

SOLVED EXAMPLES

Problem 1.

A 100-m long train is running at the speed of 30 km / hr. Find the time taken by it to pass a man standing near the railway line.

Sol.

Speed of the train $= \left(30 \times \frac{5}{18}\right)$ m/sec.$= \left(\frac{25}{3}\right)$ m/sec

Distance moved in passing the standing man = 100 m.

Required time taken $= \frac{100}{\left(\frac{25}{3}\right)} = \left(100 \times \frac{3}{25}\right)sec=$ 12sec.

Problem 2.

A train is moving at a speed of 132 km / hr. If the length of the train is 110-m, how long will it take to cross a railway platform 165-m long ?

Sol.

Speed of train $= \left(132 \times \frac{5}{18}\right)$ m/sec.$= \left(\frac{110}{3}\right)$ m/sec.

Distance covered in passing the platform = (110 + 165) m = 275 m.

∴ Time taken $= \left(275 \times \frac{3}{110}\right)$ sec

$= \frac{15}{2}$sec

$= 7\frac{1}{2}$ sec.

Problem 3.

A 160-m long train crosses a 160-m long platform in 16 seconds. Find the speed of the train.

Sol.

Distance covered in passing the platform = (160 + 160) m = 320 m.

Speed of train = $\left(\frac{320}{16}\right)$m/sec = 20 m/sec=km/hr = 72 km/hr.

Problem 4.

A person standing on a railway platform noticed that a train took 21 seconds to completely pass through the platform which was 84 m long and it took 9 seconds in passing him. Find the speed of the train in km/hr.

Sol. Let the length of the train be *x* metres.

Then, the train covers *x* metres in 9 seconds and (*x* + 84) metres in 21 seconds.

So, length of the train = 63 m.

Speed of the train = $\left(\frac{63}{9}\right)$ m/sec

= 7m/sec

= $\left(7 \times \frac{18}{5}\right)$km/hr

=$\left(\frac{126}{5}\right)$km/hr

= 25.2 km/hr.

Problem 5.

A train travelling with constant speed crosses a 90 m long platform in 12 seconds and a 120 m long platform in 15 seconds. Find the length of the train and its speed.

Sol. Let the length of the train be *x* metres.

Then, $\frac{x+90}{12} = \frac{x+120}{15} \Leftrightarrow 15\ (\mathrm{x}+\ 90)\ 12\ (\mathrm{x} + 120)$

$\Leftrightarrow 15x\ +\ 1350\ =\ 12x\ +\ 1440$

$\Leftrightarrow 3x\ =\ 90$

$\Leftrightarrow x\ =\ 30.$

Speed of the train = $\left(\frac{30+90}{12}\right)$ m/sec= 10 m/sec = $\left(10 \times \frac{18}{5}\right)$ km/hr = 36 km/hr.

Problem 6.

A 150-m long train is running with a speed of 68 kmph. In what time will it pass a man who is running at 8 kmph in the same direction in which the train is going?

Sol.

Speed of the train relative to man = $(68 - 8)$ kmph

$= \left(\mathbf{60} \times \frac{5}{18}\right)$ m/sec

$= \left(\frac{50}{3}\right)$ m/sec

Time taken by the train to cross the man

= Time taken by it to cover 150 m at $\left(\frac{50}{3}\right)$ m/sec = $\left(\mathbf{150} \times \frac{3}{50}\right)$ =sec 9 sec.

Problem 7.

A 220-m long train is running with a speed of 59 kmph. In what time will it pass a man who is running at 7 kmph in the direction opposite to that in which the train is going?

Sol.

Speed of the train relative to man = (59 + 7) kmph

$= \left(\mathbf{66} \times \frac{5}{18}\right)$ m/sec

$= \left(\frac{55}{3}\right)$ m/sec

Time taken by the train to cross the man

= Time taken by it to cover 220 m at $\left(\frac{55}{3}\right)$ sec

$= 17$ sec

Problem 8

A 300-m long train passed a man walking along the line in the same direction at the rate of 3 km/hr in 33 seconds. Find the speed of the train in km/hr.

Sol.

Speed of the train relative to man = $\left(\frac{300}{33}\right)$ m/s

$= \left(\frac{100}{11}\right)$ m/s

$= \left(\frac{100}{11} \times \frac{18}{5}\right)$ km/hr

$= \left(\frac{360}{11}\right)$ km/hr

Let the speed of the train be x km/hr. Then, relative speed = $(x - 3)$ km/hr.

$\therefore\ x - 3 = \frac{360}{11} \Leftrightarrow x = \frac{360}{11} + 3 = \frac{393}{11} = 35\frac{8}{11}$.

Hence, speed of train $= 35\frac{8}{11}$ km/hr.

Problem 9.

Two trains 100 metres and 120 metres long are running in the same direction with speeds of 72 km/hr and 54 km/hr. In how much time will the first train cross the second?

Sol.

Relative speed of the trains = $(72 - 54)$km/hr

$=$ 18 km/hr

$= \left(18 \times \frac{5}{18}\right)$ m/sec

$= 5\ m/sec$

Time taken by the trains to pass each other

= Time taken to cover (100 + 120) m at 5 m/sec

$= \left(\frac{220}{5}\right)$ sec

$= 44\ sec$

Problem 10.

A 100 m long train, takes$7\frac{1}{5}$ *seconds to cross a man walking at the rate of 5 km/hr in the direction opposite to that of the train. What is the speed of the train?*

Sol.

Let the speed of the train be x km/hr.

Speed of the train relative to man = $(x + 5)$ km/hr

$= \left[(x+5) \times \frac{5}{18}\right]$

$\therefore \frac{100}{(x+5)\times\frac{5}{18}} = \frac{36}{5}$

$\Leftrightarrow 10x + 50 = 500$

$\Leftrightarrow 10x = 450$

$\Leftrightarrow x = 45$

Hence, speed of the train = 45 km/hr.

Problem 11.

A train 100 m long travelling at 60 km/hr passes another train, twice as fast as this train and travelling in opposite direction, in 10 seconds. Find the length of the seocnd train.

Sol.

Relative speed = $(60 + 120)$ km/hr

$= \left(180 \times \frac{5}{18}\right)$ m/sec

= 50 m/sec

Let the length of the second train be *x* metres.

Then, $\frac{x+100}{10} = 50$

$\Rightarrow x + 100 = 500$

$\Rightarrow x = 400$

Hence, length of second train = 400 m.

Problem 12.

A train running at 54 kmph takes 20 seconds to pass a platform. Next it takes 12 seconds to pass a man walking at 6 kmph in the same direction in which the train is going. Find the length of the train and the length of the platform.

Sol.

Let the length of train be *x* metres and the length of platform be *y* metres.

Speed of the train relative to man = (54 – 6) kmph = 48 kmph.

$= \left(48 \times \frac{5}{18}\right)$ m/sec.

$= \left(\frac{40}{3}\right)$ m/sec.

In passing a man, the train covers its own length with relative speed.

∴ Length of train = $(Relative\ speed \times Time)$

$= \left(\frac{40}{3} \times 12\right)$m

$= 160$ m

Also, speed of the train = $\left(54 \times \frac{5}{18}\right)$ m/sec

$= 15$ m/sec.

$\therefore \left(\frac{x+y}{15}\right) = 20 \Leftrightarrow x + y = 300$

$\Leftrightarrow y = (300 - 160)\text{m} = 140$ m

Problem 13.

A moving train, 66 metres long, overtakes another train 88 metres long, moving in the same direction, in 0.168 minutes. If the second train is moving at 30 km per hour, at what speed is the first train moving?

Sol.

Let the speed of the first train be *x* km/hr.

Then, sum of lengths of trains = $(66 + 88)\,m = 154\,m$.

Relative speed of two trains = $(x - 30)kmph = \left[(x - 30) \times \frac{5}{18}\right]$ m/sec.

$\therefore \frac{154}{(x-30)\times\frac{5}{18}} = 0.168 \times 60$

$\Leftrightarrow 5(x - 30) = \frac{154\times18}{10.08} = 275$

$\Leftrightarrow x - 30 = 55$

$\Leftrightarrow x = 85.$

Hence, speed of the first train = 85 km/hr.

Problem 14.

A man sitting in a train which is travelling at 50 kmph observes that a goods train, travelling in opposite direction, takes 9 seconds to pass him. If the goods train is 280 m long, find its speed.

Sol.

Relative speed = $\left(\frac{280}{9}\right)$m/sec

$= \left(\frac{280}{9} \times \frac{18}{5}\right)$ kmph.

$= 112\ kmph.$

$\therefore$Speed of goods train = $(112 - 50)$ kmph = 62 kmph.

EXERCISE

1. A train moves with a speed of 108 kmph. Find its speed in metres per second.
2. A man sitting in a train is counting the pillars of electricity. The distance between two pillars is 60 metres, and the speed of the train is 42 km/hr. In 5 hours, how many pillars will he count?
3. In what time will a train 100 metres long cross an electric pole, if its speed be144 km / hr?
4. A 120 metre long train is running at a speed of 90 km/hr. It will cross a railway platform 230 m long in how many seconds?
5. A train covers a distance of 12 km in 10 minutes. If it takes 6 seconds to pass a telegraph post, then what is the length of the train?
6. A 50-metre long train passes over a bridge at the speed of 30 km per hour. If it takes 36 seconds to cross the brige, what is the length of the bridge?
7. A train running at the speed of 60 kmph crosses a 200 m long platform in 27 seconds. What is the length of the train?
8. A train of length 150 metres takes 40.5 seconds to cross a tunnel of length 300 metres. What is the speed of the train in km/hr?
9. A train speeds past a pole in 20 seconds and speeds past a platform 100 metres in length in 30 seconds. What is the length of the train?
10. A train running at a certain speed takes 20 seconds to cross a signal post and 50 seconds to cross a bridge. Which of the following statements is correct about the length of the bridge?
11. A jogger running at 9 kmph alongside a railway track is 240 metres ahead of the engine of a 120 metre long train running at 45 kmph in the same direction. In how much time will the train pass the jogger?
12. Two trains are moving in opposite directions @ 60 km / hr and 90 km / hr. Their lengths are 1.10 km and 0.9 km respectively. What is the time taken by the slower train to cross the faster train in seconds?
13. Two trains of equal length are running on parallel lines in the same di rect ion at 46 km / hr and 36 km / hr. The faster train passes the slower train in 36 seconds. Find the length of the length of each train.

14. Two trains of equal lengths take 10 seconds and 15 seconds respectively to cross a telegraph post. If the length of each train be 120 metres, in what time (in seconds) will they cross each other travelling in opposite direction?
15. A 150 m long train crosses a milestone in 15 seconds and a train of same length coming from the opposite direction in 12 seconds. Find the speed of the of the train.
16. Two trains running in opposite directions cross a man standing on the platform in 27 seconds and 17 seconds respectively and they cross each other in 23 seconds. Find the ratio of their speeds.
17. Two trains start at the same time form A and B and proceed toward each other at the speed of 75 km/hr and 50 km/hr respectively. when both meet at a point in between, one train was found to have travelled 175 km more than the other. Find the distance between A and B.
18. Train A passes a lamp post in 9 seconds and 700 meter long platform in 30 seconds. How much time will the same train take to cross a platform which is 800 meters long? (in seconds).

ANSWERS.

(1). 30 m/sec	**(2).** 3501	(3). 2.5 seconds	(4). 14 seconds	(5).120m
(6).250metres	**(7).** 250 metres	(8). 40	(9). 200 m	(10).5
(11). 36 sec	(12). 48	(13). 50 m	(14). 12	
(15).54kmph	(16). 3 : 2	(17). 875 km	(18). 33	

UNIT - 3

PROBLEMS ON AGES

SOLVED PROBLEMS.

Problem 1. *The ratio of the ages of Tina and Rakesh is 9 : 10 respectively. Ten years ago, the ratio of their ages was 4 : 5 respectively. What is the present age of Rakesh?*

Sol.

Let Tina's age be $9x$ years. Then, Rakesh's age = $10x$ years.

$\therefore \frac{9x-10}{10x-10} = \frac{4}{5}$

$$\Rightarrow 5(9x - 10) = 4(10x - 10) \Rightarrow 45x - 40x = 50 - 40 \Rightarrow 5x = 1 \Rightarrow x = 2.$$

$\therefore$ Present age of Rakesh = (10×2) years = 20 years.

Problem 2. Samir's age is one-fourth of his father's age and two-third of his sister Reema's age. What is the ratio of the ages of Samir, Reema and their father respectively?

Sol. Samir's age $= \frac{1}{2} \times$ Father's age $= \frac{2}{3} \times$ Reema's age = years (say).

Then, Samir's age $= x$ years, Reema's age $= \frac{3}{2}x$ years and Father's age = $4x$ years.

Ratio of the ages of Samir, Reema and father $= x : \frac{3}{2}x : 4x = 2 : 3 : 8.$

Problem 3. The age of father 10 years ago was thrice the age of his son. 10 years hence father's age will be twice that of his son. Find the ratio of their present ages.

Sol. Let son's age 10 years ago be x years. Then, father's age 10 years ago $= 3x$ years.

Son's age 10 years hence $= (x + 20)$ years.

Father's age 10 years hence = $(3x + 20)$ years.

$$\therefore\ 3x + 20 = 2(x + 20) \Rightarrow x = (40 - 20) = 20.$$

Ratio of father's age and son's age at present = $(3x + 10) : (x + 10)$

$$= (3 \times 20 + 10) : (20 + 10) = 70 : 30 = 7 : 3.$$

Problem 4. A man's present age is two-fifths of the age of his mother. After 8 years, he will be one-half of the age of his mother. How old is the mother at present?

Sol. Let mother's age be *x* years. Then, man's age = $\frac{2x}{5}$ years.

$$\frac{2x}{5} + 8 = \frac{1}{2}(x + 8) \Rightarrow \frac{2x}{5} + 8 = \frac{1}{2}x + 4 \Rightarrow \frac{1}{2}x - \frac{2x}{5} = 4$$

$$\Rightarrow 5x - 4x = 40 \Rightarrow x = 40$$

∴Mother's age = 40 years.

Problem 5. The ages of two persons differ by 16 years. If 6 years ago, the elder one be three times as old as the younger one, find their present ages.

Sol. Let their ages be *x* years and $(x - 16)$ years. Then,

$$(x - 6) = 3\{(x - 16) - 6\} \Rightarrow x - 6 = 3(x - 22) \Rightarrow 3x - x = 66 - 6 \Rightarrow 2x = 60$$
$$\Rightarrow x = 30.$$

So, their ages are 30 years and $(30 - 16) = 14$ years.

Problem 6. The product of the ages of Ankit and Nikita is 240. If twice the age of Nikita is more than Ankit's age by 4 years, then find Nikita's age.

Sol. Let Ankit's age be x years. Then, Nikita's age = $\frac{240}{x}$ years.

$$2 \times \frac{240}{x} - x = 4 \Rightarrow \frac{480}{x} - x = 4 \Rightarrow 480 - x^2 = 4x$$

$$\Rightarrow x2 + 4x - 480 = 0$$

$$\Rightarrow x2 + 24x - 20x - 480 = 0 \Rightarrow x(x + 24) - 20(x + 24) = 0$$

$$\Rightarrow (x + 24)(x - 20) = 0 \Rightarrow x = 20 \qquad [Q\ x \neq -24]$$

∴ Nikita's age = $\frac{240}{20}$ years = 12 years.

Problem 7. Reenu's age after 6 years will be three–sevenths of her father's age. 10 years ago, the ratio of their ages was 1 : 5. What is Reenu's father's age at present?

Sol. Let Reenu's age 10 years ago be *x* years.

Then, her father's age 10 years ago = $5x$ years.

$$(x + 10) + 6 = \frac{3}{7} \times [5x + 10) + 6] \Rightarrow \mathrm{x} + 16 = \frac{3}{7}(5x + 16)$$

$$\Rightarrow 7x + 112 = 15x + 48 \Rightarrow 8x = 64 \Rightarrow x = 8$$

∴ Reenu's father's present age = $(5x + 10) = (5 \times 8 + 10)$ years = 50 years.

Problem 8. The ratio of the present ages of a mother and her daughter is 7 : 1. Four years ago, the ratio of their ages was 19 : 1. What will be the mother's age four years from now?

Sol. Let mother's age be $7x$ years. Then, daughter's age = x years.

$\frac{7x-4}{x-4} = \frac{19}{1}$

$\Rightarrow 7x - 4 = 19(x - 4)$

$\Rightarrow 19x - 7x = 76 - 4$

$\Rightarrow 12x = 72$

$\Rightarrow x = 6.$

Mother's age after 4 years = $(7x + 4)$

= $(7 \times 6 + 4)$ years = 46 years.

Problem 9. The present ages of Amit and his father are in the ratio 2 : 5 respectively. Four years hence, the ratio of their ages becomes 5 : 11 respectively. What was the father's age five years ago?

Sol. Let Amit's age be $2x$ years. Then, his father's age $= 5x$ years.

$\therefore \frac{2x+4}{5x+4} = \frac{5}{11}$

$\Rightarrow 11(2x + 4) = 5(5x + 4)$

$\Rightarrow 22x + 44 = 25x + 20$

$\Rightarrow 3x = 24$

$\Rightarrow x = 8.$

Father's age 5 years ago $= (5x - 5)$ years

$= (5 \times 8 - 5)$ years $= 35$ years.

Problem 10. The ages of Shakti and Kanti are in the ratio of 8 : 7 respectively. After 10 years, the ratio of their ages will be 13 : 12. What is the difference between their ages ?

Sol. Let Shakti's age be $8x$ years. Then, Kanti's age = $7x$ years.

$\therefore \frac{8x+10}{7x+10} = \frac{13}{12}$

$\Rightarrow 12(8x + 10) = 13(7x + 10)$

$\Rightarrow 96x + 120 = 91x + 130$

$\Rightarrow 5x = 10$

$\Rightarrow x = 2.$

Difference between their ages $= (8x - 7x)$ years

$= x$ years $= 2$ years.

Problem 11. Farah got married 8 years ago. Today her age is $1\frac{2}{7}$ times her age at the time of her marriage. At present her daughter's age is one-sixth of her age. What was her daughter's age 3 years ago?

Sol. Let Farah's age 8 years ago be x years. Then, her present age = $(x + 8)$ years.

$\therefore x + 8 = \frac{9}{7}x$

$\Rightarrow 7x + 56 = 9x$

$\Rightarrow 2x = 56$

$\Rightarrow x = 28.$

$\therefore$ Farah's age now = $(x + 8)$ years = $(28 + 8)$ years = 36 years.

Her daughter's age now = $\left(\frac{1}{6} \times 36\right)$ years = 6 years.

Her daughter's age 3 years ago = (6 – 3) years = 3 years.

Problem 12. The present age of Mr. Sanyal is three times the age of his son. Six years hence, the ratio of their ages will be 5 : 2. What is the present age of Mr. Sanyal?

Sol. Let the son's age be x years. Then, Mr. Sanyal's age = $3x$ years.

$\therefore \frac{3x+6}{2x+6} = \frac{5}{2}$

$\Rightarrow 2(3x + 6) = 5(x + 6)$

$\Rightarrow 6x + 12 = 5x + 30$

$\Rightarrow x = 18.$

$\therefore$ Present age of Mr. Sanyal = $3x$ years = (3×18) years = 54 years.

Problem 13. Ratio of Rani's and Komal's ages is 3 : 5 respectively. Ratio of Komal's and Pooja's ages is 2 : 3 respectively. If Rani is two-fifth of Pooja's age, what is Rani's age?

Sol. Rani's age : Komal's age = $3 : 5 = \frac{3}{5} : 1$

Komal's age : Pooja's age = $2 : 3 = 1:\frac{3}{2}$

Rani's age : Komal's age : Pooja's age = $\frac{3}{5}:1:\frac{3}{2} = 6:10:15$

Let Rani's age be $6x$ years. Then, Komal's age = $10x$ years and

Pooja's age = $15x$ years.

Rani's age = $\frac{2}{5}$ of Pooja's age ⇒ $6x = \frac{2}{5} \times 15x$

Thus, we can not find the value of x and therefore of $6x$.

So, the answer cannot be determined.

Problem 14. The ratio between the ages of Ram and Mohan is 4 : 5 and that between the ages of Ram and Anil is 5 : 6. If the sum of the ages of the three is 69 years, what is Mohan's age?

Sol. Ram's age : Mohan's age = $4 : 5 = 1:\frac{5}{4}$

Ram's age : Anil's age = $5 : 6 = 1:\frac{6}{5}$

Let Ram's age be x years. Then, Mohan's age = $\frac{5x}{4}$ years.

And, Anil's age = $\frac{6x}{5}$ years.

$\therefore x + \frac{5x}{4} + \frac{6x}{5} = 69$

$\Rightarrow 20x + 25x + 24x = 1380$

$\Rightarrow 69x = 1380$

$\Rightarrow x = 20.$

Mohan's age = $\frac{5x}{4}$ years = $\frac{5\times20}{4}$ years = 25 years.

Problem 15. The difference between the present ages of Arun and Deepak is 14 years. Seven years ago, the ratio of their ages was 5 : 7 respectively. What is Deepak's present age?

Sol. Let the ages of Arun and Deepak 7 years ago be $5x$ years and $7x$ years respectively. Then,

Arun's present age = $(5x + 7)$ years, Deepak's present age

$= (7x + 7)$ years.

$\therefore (7x + 7) - (5x + 7) = 14$

$\Rightarrow 2x = 14$

$\Rightarrow x = 7.$

Deepak's present age = $(7 \times 7 + 7)$ years = 56 years.

Problem 16. The ages of Samina and Suhana are in the ratio of 7 : 3 respectively. After 6 years, the ratio of their ages will be 5 : 3. What is the difference in their ages?

Sol. Let Samina's age be 7*x* years. Then, Suhana's age = $3x$ years.

$\therefore \frac{7x+6}{3x+6} = \frac{5}{3}$

$\Rightarrow 3\,(7x + 6) = 5\,(3x + 6)$

$\Rightarrow 21x + 18 = 15x + 30$

$\Rightarrow 6x = 12$

$\Rightarrow x = 2.$

Difference in their ages = $(7x - 3x)$ years

$= 4x$ years= (4×2) years = 8 years.

Problem 17. The ages of A and B are presently in the ratio of 5 : 6 respectively. Six years hence, this ratio will become 6 : 7 respectively. What was B's age 5 years ago?

Sol. Let A's age be 5*x* years. Then, B's age = $6x$ years.

$\therefore \frac{5x+6}{6x+6} = \frac{6}{7}$

$\Rightarrow 7\,(5x + 6) = 6\,(6x + 6)$

$\Rightarrow 35x + 42 = 36x + 36$

$\Rightarrow x = 6.$

B's age 5 years ago = $(6x - 5)$ years

$= (6 \times 6 - 5)$ years $=$ 31 years.

Problem 18. The sum of the ages of a daughter and her mother is 56 years. After four years, the age of the mother will be three times that of the daughter. At present their ages are

Sol. Let daughter's age be *x* years. Then, mother's age $=$ $(56 - x)$ years.

$(56 - x) + 4 = 3(x + 4)$

$\Rightarrow 60 - x = 3x + 12$

$\Rightarrow 4x = 48$

$\Rightarrow x = 12.$

$\therefore$ Daughter's age $=$ 12 years, Mother's age $=$ 44 years.

Problem 19. Ram's son's age is $\frac{1}{3}$ of Ram's wife's age. Ram's wife's age is $\frac{4}{5}$ of Ram's age and Ram's age is $\frac{3}{5}$ of Ram's father's age. Find the age of Ram's son, if Ram's father is 50 years old.

Sol. Ram's father's age $=$ 50 years,

Ram's age $= \left(\frac{3}{5} \times 50\right)$ years $=$ 30 years.

Ram's wife's age $= \left(\frac{4}{5} \times 30\right)$ years $=$ 24 years.

Ram's son's age $= \left(\frac{1}{3} \times 24\right)$ years $=$ 8 years.

Problem 20. Rajan got married 8 years ago. His present age is $\frac{6}{5}$ times his age at the time of his marriage. Rajan's sister was 10 years younger to him at the time of his marriage. The age of Rajan's sister is

Sol. Let Rajan's age 8 years ago be *x* years. His present age $=$ $(x + 8)$ years.

$\therefore x + 8 = \frac{6}{5}x \Rightarrow 5x + 40 = 6x$

$\Rightarrow x = 40$

Rajan's sister's age 8 years ago = $(40 - 10)$ years = 30 years.

His sister's age now = $(30 + 8)$ years = 38 years.

Problem 21. A couple has a son and a daughter. The age of the father is four times that of the son and the age of the daughter is one-third of that of her mother. The wife is 6 years younger to her husband and the sister is 3 years older than her brother. The mother's age is

Sol. M–Mother, F–Father, S–Son and D–Daughter.

$F = 4S, D = \frac{1}{3}M,$

$M = F - 6$ and $S = D - 3$

$\therefore M = 3D = 3(S + 3)$

$= 3S + 9 = \frac{3}{4}F + 9 = \frac{3}{4}(M + 6) + 9$

$= \frac{3}{4}M + \frac{3}{4} \times 6 + 9$

$\Rightarrow \left(M - \frac{3}{4}M\right) = \left(\frac{9}{2} + 9\right) \Rightarrow \frac{1}{4}M = \frac{27}{2}$

$\Rightarrow M = \left(\frac{27}{2} \times 4\right) = 54$ years

$\therefore$ The mother is 54 years old.

Problem 22. In 10 years, A will be twice as old as B was 10 years ago. If A is now 9 years older than B, the present age of B is

Sol. Let B's age be *x* years. Then, A's age = $(x + 9)$ years.

$(x + 9) + 10 = 2(x - 10)$

$\Rightarrow x + 19 = 2x - 20$

$\Rightarrow x = 39.$

B's age = 39 years.

Problem 23. The sum of the ages of 5 children born at the intervals of 3 years each is 50 years. What is the age of the youngest child?

Sol. Let the ages of children be $x, (x + 3), (x + 6), (x + 9)$ and $(x + 12)$ years.

Then, $x + x + 3 + x + 6 + x + 9 + x + 12 = 50$

$\Rightarrow 5x = 20 \Rightarrow x = 4.$

∴ Age of youngest child = 4 years.

Problem 24. The sum of the ages of Jayant, Prem and Paras is 93 years. Ten years ago, the ratio of their ages was 2 : 3 : 4. What is the present age of Paras?

Sol. Let their ages 10 years ago be $2x$ years, $3x$ years and $4x$ years respectively.

Then, $(2x + 10) + (3x + 10) + (4x + 10) = 93$

$\Rightarrow 9x + 30 = 93$

$\Rightarrow 9x = 63 \Rightarrow x = 7.$

Present age of Paras $= (4 \times 7 + 10)$ years $=$ 38 years.

Problem 25. The ratio of a man's age and his son's age is 7 : 3 and the product of their ages is 756. The ratio of their ages after 6 years will be

Sol. Let the man's age be $7x$ years. Then, son's age $= 3x$ years.

$\therefore 7x \times 3x = 756$

$\Rightarrow 21x^2 = 756$

$\Rightarrow x^2 = 36 = 62$

$\Rightarrow x = 6.$

The ratio of their ages after 6 years $= (7x + 6) : (3x + 6)$

$= (7 \times 6 + 6) : (3 \times 6 + 6)$

$= 48 : 24 = 2 : 1.$

Problem 26. The ratio between the present ages of A and B is 5 : 3 respectively. The ratio between A's age 4 years ago and B's age 4 years hence is 1 : 1. What is the ratio between A's age 4 years hence and B's age 4 years ago?

Sol. Let *A'*s age be $5x$ years. Then, B's age = $3x$ years.

$\frac{5x-4}{3x+4} = \frac{1}{1} \Rightarrow 5x - 4 = 3x + 4 \Rightarrow 2x = 8 \Rightarrow x = 4$

$\therefore \frac{A's\ age\ 4\ years\ hence}{B's\ age\ 4\ years\ ago} = \frac{5x+4}{3x-4}$

$$= \frac{5\times4+4}{3\times4-4} = \frac{24}{8} = \frac{3}{1} = 3:1$$

Problem 27. The ratio between the ages of Neelam and Shiny is 5 : 6 respectively. If the ratio between the one-third age of Neelam and half of Shiny's age is 5 : 9, then what is Shiny's age?

Sol. Let Neelam's age be $5x$ years and Shiny's age be $6x$ years.

$$\left(\frac{1}{3} \times 5x\right) : \left(\frac{1}{2} \times 6x\right) = 5:9 \Rightarrow \frac{5x}{3 \times 3x} = \frac{5}{9}$$

Thus, Shiny's age cannot be determined.

Problem 28. A man is aged three times more than his son Ronit. After 8 years, he would be two and a half times of Ronit's age. After further 8 years, how many times would he be of Ronit's age?

Sol. Let Ronit's present age be x years.

Then, the man's age = $(x + 3x)$ years = $4x$ years.

$4x + 8 = \frac{5}{2}(x + 8) \Rightarrow 8x + 16 = 5x + 40$

$\Rightarrow 3x = 24 \Rightarrow x = 8$

$\therefore$ Required ratio $= \frac{(4x+16)}{(x+16)} = \frac{(4\times8+16)}{(8+16)} = \frac{48}{24} = 2$ times.

Problem 29. The age of a man 10 years ago was thrice the age of his son. 10 years hence, the man's age will be twice the age of his son. The ratio of their present ages is

Sol. Let son's age 10 years ago be x years. Then, man's age 10 years ago $= 3x$ years.

Son's present age $= (x + 10)$ years, Man's present age $= (3x + 10)$ years.

$(3x + 10) + 10 = 2(x + 10 + 10)$

$\Rightarrow 3x + 20 = 2(x + 20)$

$\Rightarrow 3x + 20 = 2x + 40$

$\Rightarrow x = 20.$

Ratio of present ages of man and the son

$$= \frac{3x+10}{x+10} = \frac{3\times20+10}{20+10} = \frac{70}{30} = 7:3$$

Problem 30. The difference between the ages of two men is 10 years. 15 years ago, the elder one was twice as old as the younger one. The present age of the elder man is

Sol. Let their ages be x years and $(x + 10)$ years.

Then $(x + 10 - 15) = 2(x - 15)$

$\Rightarrow x - 5 = 2x - 30$

$\Rightarrow x = 25.$

Present age of the elder man $= (x + 10)$ years $= (25 + 10)$ years $= 35$ years.

Problem 31. Sneh's age is $\frac{1}{6}th$ of her father's age. Sneh's father's age will be twice of Vimal's age after 10 years. If Vimal's 8th birthday was celebrated 2 years ago, then what is Sneh's present age?

Sol. Vimal's present age $= (8 + 2)$ years $= 10$ years.

Sneh's father's age $= 2(10 + 10)$ years $= 40$ years.

Sneh's age $= \left(\frac{1}{6} \times 40\right)$ years $= \frac{20}{3}$ years $6\frac{2}{3}$ years.

Problem 32. The ages of Sulekha and Arunima are in the ratio 9 : 8 respectively. After 5 years, the ratio of their ages will be 10 : 9. What is the difference in their ages?

Sol. Let Sulekha's age be $9x$ years. Then, Arunima's age = $8x$ years.

$\frac{9x+5}{8x+5} = \frac{10}{9}$

$\Rightarrow 9\,(9x + 5) = 10\,(8x + 5)$

$\Rightarrow 81x + 45 = 80x + 50$

$\Rightarrow x = 5.$

Difference in their ages = $(9x - 8x)$ years = x years = 5 years.

Problem 33. If 10 years are subtracted from the present age of Mr. Roy and the remainder divided by 14, then you would get the present age of his grandson Sachin. If Sachin is 9 years younger to Saloni whose age is 14 years, then what is the present age of Mr. Roy?

Sol. Saloni's age = $14\ years \Rightarrow$ Sachin's age = $(14 - 9)$ years = 5 years.

Let the present age of Mr. Roy be x years.

$\frac{x-10}{14} = 5 \Rightarrow x - 10 = 70$

$\Rightarrow x = 80$ years

Problem 34. Eight year ago, Poorvi's age was equal to the sum of the present ages of her one son and one daughter. Five years hence, the respective ratio between the ages of her daughter and her son that time will be 7 : 6. If Poorvi's husband is 7 years elder to her and his present age is three times the present age of their son, what is the present age of the daughter?

Sol. Let the age of the son and the daughter of Poorvi be $6a$ years and $7a$ years respectively. 5 years hence, present age of son = $6a - 5$ and present age of daughter = $7a - 5$

According to the question,

Eight years ago, the age of Poorvi = $6a - 5 + 7a - 5$

$= 13a - 10$

So, present age of Poorvi = $13a - 10 + 8 = 13a - 2$.

Since, present age of Poorvi husband = $3(6a - 5)$

The difference of present age of Poorvi husband and

Poorvi = 7, (given)

$3(6a - 5) - (13a - 2) = 7, \Rightarrow 18a - 15 - 13a + 2 = 7$

$\Rightarrow 5a = 20 \Rightarrow a = 4$

The present age of daughter = $(7a - 5) = 7 \times 4 - 5 = 23$ years

Problem 35. Rahul is as much younger than Sagar as he is older than Purav. If the sum of the ages of Purav and Sagar is 66 years, and Sagar's age is 48 years, then what is Purav's age? (in years)

Sol. Let the age of Rahul, Sagar and Purav be *x, y* and *z* respectively

According to the given information

Age of Sagar – Age of Rahul = Age of Rahul – Age of Purav

$\Rightarrow y - x = x - z$

$\Rightarrow 2x = y + z \ldots (i)$

Also $y + z = 66$ years

From (i) $x = 33$ years

Also as per Eq (i) we have Purav's age + Sagar's age = 66 years.

By going through option (a) given Purav = 18, and

Rahul = 33 years, Sagar = 48 years

Difference between Rahul's and Purav's age = 18 years.

Problem 36. Ten years hence, the respective ratio between Simmi's age and Niti's age will be 7 : 9. Two years ago, the respective ratio between Simmi's age and Niti's age was 1 : 3. If Abhay is 4 years older to his sister Niti, what is Abhay's present age? (in years)

Sol. Let present ages of Simmi and Niti be *a* and *b* years, respectively.

Ten years hence, the ratio between Simmi's age and Niti's age = $7 : 9$

According to the question, $\frac{a+10}{b+10} = \frac{7}{9}$

By cross multiplying we get

$\Rightarrow 9a + 90 = 7b + 70$

$\Rightarrow 7b - 9a = 20 \; \ldots..(i)$

Also, $\frac{a-2}{b-2} = \frac{1}{3}$

By cross multiplying we get

$3a - 6 = b - 2$

$\Rightarrow 3a - b = 4 \; \ldots(ii)$

Multiplying equation (ii) by 3

$9a - 3b = 12 \ldots(iii)$

Adding equation (ii) and (iii) we get

$-9a + 7b = 20$

$4b = 32 \Rightarrow b = 8 year$

From equation (ii) we get

$a = \frac{4+b}{3} = \frac{4+8}{3} = \frac{12}{3} = 4$ years

Since, Abhay is 4 years older to Niti.

So, Abhay present age = $8 + 4 = 12$ years.

Exercise

1. The ages of Nishi and Vinnee are in the ratio 6 : 5 respectively. After 9 years, the ratio of their ages will be 9 : 8. What is the difference in their ages now?
2. The ratio of the ages of a father and his son is 17 : 7 respectively. Six years ago, the ratio of their ages was 3 : 1 respectively. What is the father's present age?
3. The ages of A and B are in the ratio 6 : 5 and the sum of their ages is 44 years. What will be the ratio of their ages after 8 years?
4. The age of a mother today is thrice that of her daughter. After 12 years, the age of the mother will be twice that of her daughter. The present age of the daughter is:
5. The average of the ages of a man and his daughter is 34 years. If the respective ratio of their ages four years from now is 14 : 5, what is daughter's present age?
6. The age of a father 10 years ago was thrice the age of his son. 10 years hence, the father's age will be twice that of his son. The ratio of their present ages is
7. At present, Suresh's age is twice the age of his daughter. After 6 years from now, the ratio of the ages of Suresh and his daughter will be 23 : 13. What is the present age of Suresh?
8. Ten years ago, a man was seven times as old as his son. Two years hence, twice his age will be equal to five times the age of his son. What is the present age of the son?
9. The ages of Sulekha and Arunima are in the ratio of 9 : 8 respectively. After 5 years, the ratio of their ages will be 10 : 9. What is the difference in their ages?
10. The age of the mother today is thrice that of her daughter. After 12 years, the age of the mother will be twice that of her daughter. The age of the daughter today is
11. The present age of son is half of the present age of his mother. Ten years ago, his mother's age was thrice the age of her son. What is the present age of the son?
12. Ratio between the ages of Subhash, Prasad and Amar is 3 : 6 : 7. If the difference between the ages of Prasad and Amar is 10 years, then what is the difference between the ages of Subhash and Prasad?
13. The ages of two persons differ by 20 years. If 5 years ago, the older one be 5 times as old as the younger one, then their present ages are
14. The present ages of three persons are in the proportion 4 : 7 : 9. Eight years ago, the sum of their ages was 56 years. The present age of the eldest person is

15. Reenu's father was 38 years of age when she was born while her mother was 36 years old when her brother 4 years younger to her was born. What is the difference between the ages of her parents?
16. A man was asked to state his age in years. His reply was, "Take my age 3 years hence, multiply it by 3 and then subtract 3 times my age 3 years ago and you will know how old I am." What is the age of the man?
17. The sum of the ages of a man and his son is 45 years. Five years ago, the product of their ages was 34. The man's age is
18. Sonal is 40 years old and Nitya is 60 years old. How many years ago was the ratio of their ages 3 : 5?
19. The ratio of the ages of a man and his wife is 4 : 3. After 4 years, this ratio will be 9 : 7. If at the time of their marriage, the ratio of their ages was 5 : 3, then how many years ago were they married?
20. 18 years ago, a man was three times as old as his son. Now, the man is twice as old as his son. The sum of the present ages of the man and his son is
21. One year ago, Promila was four times as old as her daughter Sakshi. Six years hence, Promila's age will exceed her daughter's age by 9 years. The ratio of the present ages of Promila and her daughter is
22. Tanya's grandfather was 8 times older to her 16 years ago. He would be 3 times of her age 8 years from now. 8 years ago, what was the ratio of Tanya's age to that of her grandfather?
23. 6 years ago, the ratio of the ages of Kunal and Sagar was 6 : 5. Four years hence, the ratio of their ages will be 11 : 10. What is Sagar's age at present?
24. The ages of Samina and Suhana are in the ratio of 7 : 3 respectively. After 6 years, the ratio of their ages will be 5 : 3. What is the difference in their ages?
25. Three years ago, the ratio of the ages of Amisha and Nimisha was 8 : 9 respectively. 3 years hence, the ratio of their ages will be 11 : 12respectively. What is the present age of Amisha?
26. X's age 3 years ago was three times the present age of Y. At present, Z's age is twice the age of Y. Also Z is 12 years younger than X. What is the present age of Z?
27. The sum of present ages of a father and his son is 8 years more than the present age of the mother. The mother is 22 years older than the son. What will be the age of the father after 4 years?

28. 4 years ago, the ratio of 1/2 of A's age at that time and four times of B's age at the time was 5 : 12. Eight years hence, 1/2 of A's age at that time will be less than B's age at that time by 2 years. What is B's present age?

UNIT - 4

AREA

FUNDAMENTAL CONCEPTS

I. Results on Triangles:

1. Sum of the angles of a triangle is 180°.

2. The sum of any two sides of a triangle is greater than the third side.

3. Pythagoras' Theorem: In a right-angled triangle, (Hypotenuse)2 = (Base)2 + (Height)2

4. The line joining the mid-point of a side of a triangle to the opposite vertex is called the median.

5. The point where the three medians of a triangle meet, is called centroid. The centroid divides each of the medians in the ratio 2 : 1.

6. In an isosceles triangle, the altitude from the vertex bisects the base.

7. The median of a triangle divides it into two triangles of the same area.

8. The line joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.

9. The four triangles formed by joining the mid-points of the sides of a given triangle are equal in area, each equal to one-fourth of the given triangle.

10. The ratio of the areas of two similar triangles is equal to the ratio of the squares of their

(i) corresponding sides (ii) corresponding altitudes

II. Results on Quadrilaterals:

1. The diagonals of a parallelogram bisect each other.

2. Each diagonal of a parallelogram divides it into two triangles of the same area.

3. The diagonals of a rectangle are equal and bisect each other.

4. The diagonals of a square are equal and bisect each other at right angles.

5. The diagonals of a rhombus are unequal and bisect each other at right angles.

6. A parallelogram and a rectangle on the same base and between the same parallels are equal in area.

7. Of all the parallelograms of given sides, the parallelogram which is a rectangle has the greatest area.

8. The line joining the mid-points of the non-parallel sides of a trapezium is parallel to each of the parallel sides and equal to half of their sum.

9. The line joining the mid-points of the diagonals of a trapezium is parallel to each of the parallel sides and equal to half of their difference.

IMPORTANT FORMULAE

I. 1. Area of a rectangle = (Length × Breadth).

∴ Length $= \left(\frac{Area}{Breadth}\right)$ and Breadth$=\left(\frac{Area}{\text{Length}}\right)$

2. Perimeter of a rectangle = 2 (Length + Breadth).

II. Area of a square $= (side)^2 = \frac{1}{2}(diagonal)^2$.

III. Area of 4 walls of a room = 2 (Length + Breadth) × Height.

IV. 1. Area of a triangle$= \frac{1}{2} \times Base \times Height$.

2. Area of a triangle = $\sqrt{s\,(s-a)\,(s-b)\,(s-c)}$, where a, b, c are the sides of the triangle and $s = \frac{1}{2}(a+b+c)$.

3. Area of an equilateral triangle $= \frac{\sqrt{3}}{4} \times (sides)^2$

4. Area of a triangle $= \frac{1}{2} ab \sin\theta$, where a and b are the lengths of any two sides of the triangle and θ is the angle between them.

5. Radius of incircle of an equilateral triangle of side $a = \frac{a}{2\sqrt{3}}$.

6. Radius of circumcircle of an equilateral triangle of side $a = \frac{a}{\sqrt{3}}$.

7. Radius of incircle of a triangle of area Δ and semi-perimeter $s = \frac{\Delta}{s}$

8. Radius of circumcircle of a triangle $= \frac{Product\ of\ sides}{4\Delta}$.

V. 1. Area of a parallelogram = (Base × Height).

2. Area of a rhombus $= \frac{1}{2} \times (Product\ of\ diagonals)$.

3. Area of a trapezium $= \frac{1}{2}\ (sum\ of\ parallel\ sides) \times (distance\ between\ them)$

VI. 1. Area of a circle $= \pi R^2$, where R is the radius.

2. Circumference of a circle $=\ 2\pi R$.

3. Length of an arc $= \frac{2\pi R\theta}{360}$, where θ is the central angles

4. Area of a sector $= \frac{1}{2}(arc\ length \times R) = \frac{\pi R^2\theta}{360}$.

VII. 1. Area of a semi-circle $= \frac{\pi R^2}{2}$

2. Circumference of semi-circle $= \pi R$.

3. Perimeter of a semi-circle $= \pi R\ +\ 2R$.

VIII. 1. Area of a regular polygon of N sides, with a as the length of each side $= \frac{a^2N}{4\tan\left(\frac{180}{N}\right)}$

2. Area of a regular hexagon of side $a\ = \frac{3}{2}\sqrt{3}\ a^2$

3. Area of a regular pentagon of side $a\ =\ 1.72\ a^2$.

4. The area enclosed between the circumcircle and incircle of a regular polygon of side $a\ =$ $\frac{\pi a^2}{4}$.

SOLVED PROBLEMS

Problem 1. Find the maximum distance between two points on the perimeter of a rectangular garden whose length and breadth are 100 m and 50 m.

Sol. Clearly, the two points which are maximum distance apart are the end-points of a diagonal.

$\therefore$ Reqd. distance = Length of the diagonal = $\sqrt{(100)^2 + (50)^2}$ m

$$= \sqrt{1000 + 2500}\, m = \sqrt{12500}\, m$$

$$= 50\sqrt{5}\, m = (50 \times 2.236) = 111.8\, m.$$

Problem 2. One side of a rectangular field is 15 m and one of its diagonals is 17 m. Find the area of the field.

Sol. Other side = $\sqrt{(17)^2 - (15)^2} = \sqrt{289 - 225} = \sqrt{64} = 8\, m.$

$\therefore$ Area = $(15 \times 8) m^2 = 120\, m^2.$

Problem 3. A lawn is in the form of a rectangle having its sides in the ratio 2 : 3. The area of the lawn is $\frac{1}{6}$ hectares. Find the length and breadth of the lawn.

Sol. Let length = $2x$ metres and breadth = $3x$ metres.

Now, area = $\left(\frac{1}{6} \times 1000\right) m^2 = \left(\frac{5000}{3}\right) m^2$

So, $2x \times 3x = \frac{5000}{3} \Leftrightarrow x^2 = \frac{2500}{9} \Leftrightarrow x = \left(\frac{50}{3}\right)$

$\therefore$ Length $= 2x = \frac{100}{3} m = 33\frac{1}{3} m$ and Breadth $= 3x = \left(3 \times \frac{50}{3}\right) m = 50m.$

Problem 4. Find the cost of carpeting a room 13 m long and 9 m broad with a carpet 75 cm wide at the rate of Rs. 12.40 per square metre.

Sol. Area of the carpet = Area of the room = $(13 \times 9) m^2 = 117\, m^2.$

Length of the carpet = $\left(\frac{Area}{Width}\right) = \left(117 \times \frac{4}{3}\right) m = 156m$

$\therefore$ Cost of carpeting = $Rs. (156 \times 12.40) = Rs. 1934.40.$

Problem 5. The length of a rectangle is twice its breadth. If its length is decreased by 5 cm and breadth is increased by 5 cm, the area of the rectangle is increased by 75 sq. cm. Find the length of the rectangle.

Sol. Let breadth = x. Then, length = $2x$. Then,

$(2x - 5)(x + 5) - 2x \times x = 75 \Leftrightarrow 5x - 25 = 75 \Leftrightarrow x = 20.$

∴ Length of the rectangle = 20 cm.

Problem 6. A rectangular carpet has an area of 120 sq. metres and a perimeter of 46 metres. Find the length of its diagonal.

Sol. Let the length and breadth of the rectangle be *l* and *b* metres respectively.

Then, $2(l + b) = 46 \Rightarrow l + b = 23 \Rightarrow b = (23 - l)$.

And, $lb = 120 \Rightarrow l(23 - l) = 120 \Rightarrow 23l - l2 = 120 \Rightarrow l2 - 23l + 120 = 0$

$\Rightarrow l2 - 15l - 8l + 120 = 0$

$\Rightarrow l(l - 15) - 8(l - 15) = 0$

$\Rightarrow (l - 15)(l - 8) = 0 \Rightarrow l = 15.$

So, $l = 15$ and $b = 8$.

∴Length of diagonal = $\sqrt{l^2 + b^2} = \sqrt{(15)^2 + 8^2}m = \sqrt{289}\,m = 17\,m.$

Problem 7. The length of a rectangle is increased by 30%. By what percent would the breadth have to be decreased to maintain the same area?

Sol. Let the length and breadth of the rectangle be *l* and *b* units respectively.

Then, area of rectangle = (lb) sq. units.

New length = 160% of $l = \frac{8l}{5}$ units.

Desired breadth = $\frac{Area}{New\ length} = \frac{lb}{\left(\frac{8l}{5}\right)} = \frac{5b}{8}$ units

Decrease in breadth = $\left(b - \frac{5b}{8}\right) units = \frac{3b}{8} units$

∴Decrease% = $\left(\frac{3b}{8} \times \frac{1}{b} \times 100\right)\% = \frac{75}{2}\% = 37.5\%$

Problem 8. In measuring the sides of a rectangular plot, one side is taken 5% in excess and the other 6% in deficit. Find the error percent in area calculated, of the plot.

Sol. Let the length and breadth of the rectangle be *l* and *b* units respectively.

Then, correct area = (lb) sq. units.

Calculated area = $\left(\frac{105l}{100} \times \frac{94b}{100}\right) = \left(\frac{987\ lb}{1000}\right)\ sq.units$

Error in measurement = $\left(lb - \frac{987}{1000}\ lb\right) sq.units = \left(\frac{13b}{1000}\right) sq.units$

∴Error% $= \left(\frac{13lb}{1000} \times \frac{1}{lb} \times 100\right)\% = 1.3\%$

Problem 9. Instead of walking along two adjacent sides of a rectangular field, a boy took a shor-cut along the diagonal of the field and saved a distance equal to half of the longer side. Find the ratio of the shorter side of the rectangle to the longer side.

Sol. Let the length of the longer side of the field be *l* and that of the shorter side be *b*.

Then, diagonal = $\sqrt{l^2 + b^2}$.

$\therefore (l + b) - \sqrt{l^2 + b^2} = \frac{1}{2}l \Rightarrow \sqrt{l^2 + b^2} = \frac{1}{2} + b.$

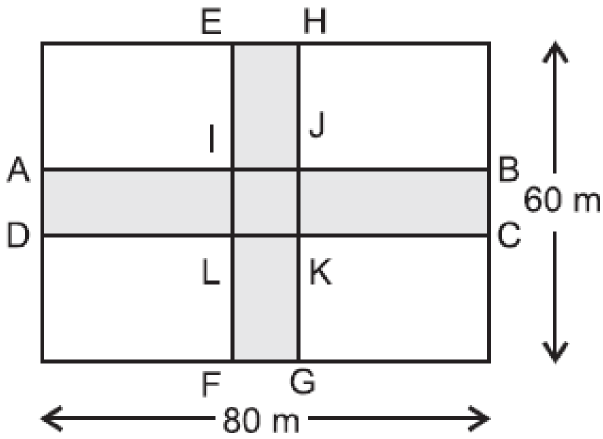
$$\Rightarrow 2\sqrt{l^2 + b^2} = l + 2b \Rightarrow 4(l^2 + b^2) = l^2 + 4b^2 + 4lb$$

$$\Rightarrow 3l^2 = 4lb \Rightarrow 3l = 4b \Rightarrow \frac{b}{l} = \frac{3}{4}$$

Hence, required ration = $3:4$

Problem 10. Two perpendicular cross roads of equal width run through the middle of a rectangular field of length 80 m and breadth 60 m. If the area of the cross roads is 675 m2, find the width of the roads.

Sol. Let *ABCD* and *EFGH* denote the cross roads, each of width *x* metres.



Then, area of the cross-roads = area of rectangle *ABCD* + area of rectangle *EFGH* – area of square *IJKL*

$$= (80x + 60x - x^2) = 140x - x^2.$$

$\therefore 140x - x^2 = 675 \Rightarrow x^2 - 140x + 675 = 0$

$\Rightarrow x^2 - 135x + 5x - 675 = 0 \Rightarrow x(x - 135) - 5(x - 135) = 0$

$\Rightarrow (x - 135)(x - 5) = 0 \Rightarrow x = 5.$ $\quad [Q\ x \neq 135]$

So, width of road = 5m.

Problem 11. A rectangular grassy plot 110 m by 65 m has a gravel path 2.5 m wide all round it on the inside. Find the cost of gravelling the path at 80 paise per sq. metre.

Sol. Area of the plot = $(110 \times 65)m^2 = 7150\ m^2$.

Area of the plot excluding the path = $[(110 - 5) \times (65 - 5)]m^2 = 6300\ m^2$.

$\therefore$ Area of the path = $(7150 - 6300)m^2 = 850\ m^2$.

Cost of gravelling the path = $Rs.\left(850 \times \frac{80}{100}\right) = Rs. 680$.

Problem 12. The diagonal of a rectangular field is 15 m and its area is 108 sq. m. What will be the total expenditure in fencing the field at the rate of Rs.5 per metre?

Sol. Let the length and breadth of the rectangle be *l* and *b* metres respectively.

Then, $\sqrt{l^2 + b^2} = 15$ and $lb = 108 \Rightarrow l^2 + b^2 = 225$ and $lb = 108$

$\Rightarrow\ (l + b)^2 = (l^2 + b^2) + 2lb = 225 + 216 = 441$

$\Rightarrow\ l + b = \sqrt{441} = 21.$

∴Perimeter of the field = $2\,(l + b) = (2 \times 21)\,m = 42\,m.$

Hence, cost of fencing = $Rs.(42 \times 5) = Rs. 210.$

Problem 13. The perimeters of two squares are 40 cm and 32 cm. Find the perimeter of a third square whose area is equal to the difference of the areas of the two squares.

Sol. Side of first square = $\left(\frac{40}{4}\right) cm = 10cm;$

Side of second square = $\left(\frac{32}{4}\right) cm = 8cm$

Area of third square = $[(10)^2 - (8)^2]cm^2 = (100 - 64)cm^2 = 36\ cm^2.$

Side of third square = $\sqrt{36}\ cm = 6\ cm.$

∴ Required perimeter = $(6 \times 4)\ cm = 24\ cm.$

Problem 14. The length of a rectangle R is 10% more than the side of a square S. The width of the rectangle R is 10% less than the side of the square S. What is the ratio of the area of R to that of S?

Sol. Let each side of the square *S* be *x* units.

Then, length of rectangle $R\ = 110\%\ of\ x = \left(\frac{11x}{10}\right)$units

And, width of rectangle $R\ = 90\%\ of\ x = \left(\frac{9x}{10}\right)$ units

∴Ratio of areas of *R* and $S\ = \left(\frac{11x}{10} \times \frac{9x}{10}\right) : x^2 = \frac{99x^2}{100} : x^2 = 99 : 100$

Problem 15. Find the largest size of a bamboo that can be placed in a square of area 100 sq. m.

Sol. Side of the square = $\sqrt{100}\ m = 10\ m.$

Largest size of bamboo = Length of diagonal of the square

$= 10\sqrt{2}\,m. = (10 \times 1.414)\,m = 14.14\,m.$

Problem 16. A rectangular courtyard, 3.78 m long and 5.25 m broad, is to be paved exactly with square tiles, all of the same size. Find the least number of square tiles covered.

Sol. Area of the room = $(378 \times 525)cm^2$.

Size of largest square tile = H.C.F. of 378 cm and 525 cm = 21 cm.

Area of 1 tile = $(21 \times 21)cm^2$.

∴Number of tiles required = $\left(\frac{378\times525}{21\times21}\right) = 450$

Problem 17. Find the area of a square, one of whose diagonals is 3.8 m long.

Sol. Area of the square = $\frac{1}{2} \times (diagonal)^2 = \left(\frac{1}{2} \times 3.8 \times 3.8\right) m^2 = 7.22m^2$

Problem 18. The diagonals of two squares are in the ratio of 2 : 5. Find the ratio of their areas.

Sol. Let the diagonals of the squares be 2*x* and 5*x* respectively.

∴ Ratio of their areas = $\frac{1}{2} \times (2x)^2 : \frac{1}{2} \times (5x)^2 = 4x^2 : 25x^2 = 4 : 25$

Problem 19. If each side of a square is increased by 25%, find the percentage change in its area.

Sol. Let each side of the square be a. Then, area = a^2.

New side = $\frac{125a}{100} = \frac{5a}{4}$.

New area = $\left(\frac{5a}{4}\right)^2 = \frac{25a^2}{16}$.

Increase in area = $\left(\frac{25a^2}{16} - a^2\right) = \frac{9a^2}{16}$

$\therefore$ Increase % $= \left(\frac{9a^2}{16} \times \frac{1}{a^2} \times 100\right)\% = 56.25\%.$

Problem 20. If the diagonal of a square is decreased by 15%, find the percentage decrease in its area.

Sol. Let the length of the diagonal of the square be x. Then, area $= \frac{x^2}{2}$

New diagonal $= 85\%\ of\ x = \frac{17x}{20}$

New area $= \frac{1}{2} \times \left(\frac{17x}{20}\right)^2 = \frac{289x^2}{800}$

Decrease in area $= \left(\frac{x^2}{2} - \frac{289x^2}{800}\right) = \frac{111x^2}{800}$

$\therefore$Decrease % $= \left(\frac{111x^2}{800} \times \frac{2}{x^2} \times 100\right)\% = 27.75\%$

Problem 21. If the side of a square is increased by 8 cm, its area increases by 120 sq. cm. Find the side of the square.

Sol. Let the length of a side of the square be *x* cm. Then,

$$(x + 8)^2 - x^2 = 120 \Rightarrow 64 + 16x = 120 \Rightarrow 16x = 56$$

$$\Rightarrow x = \frac{56}{16} = \frac{7}{2} = 35cm$$

Hence, side of square $= 3.5\ cm$.

Problem 22. If the length of a certain rectangle is decreased by 4 cm and the width is increased by 3 cm, a square with the same area as the original rectangle would result. Find the perimeter of the original rectangle.

Sol. Let *x* and *y* be the length and breadth of the rectangle respectively.

Then, $x - 4 = y + 3\ or\ x - y = 7 \ldots (i)$

Area of the rectangle $= xy$; Area of the square $= (x - 4)(y + 3)$

$$\therefore (x - 4)(y + 3) = xy \Leftrightarrow 3x - 4y = 12 \ldots (ii)$$

Solving (i) and (ii), we get $x = 16$ and $y = 9$.

$\therefore$ Perimeter of the rectangle $= 2(x + y) = [2(16 + 9)]\ cm = 50\ cm.$

Problem 23. The dimensions of a room are 12.5 metres by 9 metres by 7 metres. There are 2 doors and 4 windows in the room; each door measures 2.5 metres by 1.2 metres and each window 1.5 metres by 1 metre. Find the cost of painting the walls at Rs.36.50 per square metre.

Sol. Area of 4 walls $= 2(l + b) \times h = 2[(12.5 + 9) \times 7] m^2 = 301\ m^2$.

Area of 2 doors and 4 windows $= [2(2.5 \times 1.2) + 4(1.5 \times 1)] m^2 = 12\ m^2$.

Area to be painted $= (301 - 12) m^2 = 289\ m^2$.

$\therefore$Cost of painting = Rs. (289 × 36.50) = Rs. 10548.50.

Problem 24. A room is half as long again as it is broad. The cost of carpeting the room at Rs. 5 per sq. m is Rs. 270 and the cost of papering the four walls at Rs. 10 per m^2 is Rs. 1720. If a door and 2 windows occupy 8 sq. m, find the dimensions of the room.

Sol. Let breadth $= x$ metres, length $= \frac{3x}{2}$ metres, height $= H$ metres.

Area of the floor $= \left(\frac{Total\ cost\ of\ carpeting}{\frac{Rate}{m^2}}\right) m^2 = \left(\frac{270}{5}\right) m^2 = 54 m^2$

$\therefore x \times \frac{3x}{2} = 54 \Leftrightarrow x^2 = \left(54 \times \frac{2}{3}\right) = 36 \Leftrightarrow x = 6.$

So, breadth $= 6\ m$ and length $= \left(\frac{3}{2} \times 6\right) m = 9m$

Now, papered area $= \left(\frac{1720}{10}\right) m^2 = 172 m^2$

Area of 1 door and 2 window $= 8m^2$

Total area of 4 walls $= (172 + 8) m^2 = 180 m^2$

$\therefore 2(9 + 6) \times H = 180 \Leftrightarrow H = \left(\frac{180}{30}\right) = 6m$.

Problem 25. Find the area of a triangle whose sides measure 15 cm, 16 cm and 17cm.

Sol. Let $a = 15$ cm, $b = 16$ cm and $c = 17$ cm.

Then, $s = \frac{1}{2}(a + b + c) = 24$

∴(s – a) = 9 cm, (s – b) = 8 cm and $(s - c) = 7\ cm$.

Area = $\sqrt{s\,(s - a)(s - b)(s - c)} = \sqrt{24 \times 9 \times 8 \times 7}\ cm^2$

$$= 24\sqrt{21}\ cm^2.$$

Problem 26. Find the area of a right–angled triangle with hypotenuse 65 cm and one side 25 cm.

Sol. Other side = $(65)^2 - (25)^2\ cm = 3600\ cm = 60\ cm.$

∴Area of the triangle = $\frac{1}{2} \times$ product of sides containing the right angle

$$= \left(\frac{1}{2} \times 60 \times 25\right) cm^2 = 750cm$$

Problem 27. The base of a triangular field is three times its altitude. If the cost of cultivating the field at Rs. 24.68 per hectare be Rs. 333.18, find its base and height.

Sol. Area of the field = $\frac{Total\ Cost}{Rate} = \left(\frac{333.18}{24.68}\right) hectares = 13.5\ hectares$

$$= (13.5 \times 10000)m^2 = 135000\ m^2.$$

Let altitude = x metres and base = $3x$ metres.

Then, $\frac{1}{2} \times 3x \times x = 135000 \Leftrightarrow x^2 = 90000 \Leftrightarrow x = 300$

∴ Base = 900 m and Altitude = $300\ m$.

Problem 28. The cost of fencing an equilateral triangular park and a square park is the same. If the area of the triangular park is $16\sqrt{3}m^2$, find the length of the diagonal of the diagonal of the square park.

Sol. Let the length of each side of the triangular park be *a* cm.

Then, $\frac{\sqrt{3}}{4}a^2 = 16\sqrt{3} \Rightarrow a^2 = 64 \Rightarrow a = 8m.$

Perimeter of the square park = Perimeter of the triangular park = $(3 \times 8)m$

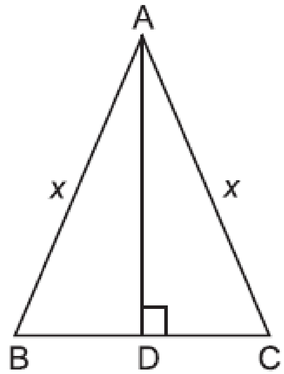
$= 24\, m.$

Side of the square park = $\left(\frac{24}{4}\right) cm = 6m$

∴Length of diagonal of the square park = $6\sqrt{2}\, m.$

Problem 29. The altitude drawn to the base of an isosceles triangle is 8 cm and the perimeter is 32 cm. Find the area of the triangle.

Sol. Let *ABC* be the isosceles triangle and *AD* be the altitude.



Let $AB = AC = x$. Then, $BC = (32 - 2x)$.

Since, in an isosceles triangle, the altitude bisects the base,

so $BD = DC = (16 - x)$.

In $\Delta\, ADC, AC^2 = AD^2 + DC^2$

$\Rightarrow x^2 = (8)^2 + (16 - x)^2$

$\Rightarrow 32x = 320 \Rightarrow x = 10.$

$\therefore BC = (32 - 2x) = (32 - 20)\, cm = 12\, cm.$

Hence, required area = $\left(\frac{1}{2} \times BC \times AD\right)$

$= \left(\frac{1}{2} \times 12 \times 10\right) cm^2 = 60cm^2.$

Problem 30. The base and altitude of a right angled triangle are 12 cm and 5 cm respectively. Find the perpendicular distance of its hypotenuse from the opposite vertex.

Sol. Area of the triangle = $\left(\frac{1}{2} \times 12 \times 5\right) cm^2 = 30cm^2$

Hypotenuse = $\sqrt{(12)^2 + 5^2}\ cm = \sqrt{169}\ cm = 13\ cm.$

Let the perpendicular distance of the hypotenuse from the opposite vertex be h cm.

Then, $\frac{1}{2} \times 13 \times h = 30 \Rightarrow h = \frac{60}{13} = 4\frac{8}{13}\ cm.$

Problem 31. In two triangles, the ratio of the areas is 4 : 3 and the ratio of their heights is 3 : 4. Find the ratio of their bases.

Sol. Let the bases of the two triangles be *x* and *y* and their heights be 3*h* and 4*h* respectively.

Then, $\frac{\frac{1}{2}\times x\times 3h}{\frac{1}{2}\times y\times 4h} = \frac{4}{3}$

$\Leftrightarrow \frac{x}{y} = \left(\frac{4}{3}\times\frac{4}{3}\right) = \frac{16}{9}.$

$\therefore$ Required ratio = $16 : 9.$

Problem 32. The base of a parallelogram is twice its height. If the area of the parallelogram is 72 sq. cm, find its height.

Sol. Let the height of the parallelogram be x cm. Then, base = $(2x)\ cm.$

$\therefore\ 2x \times x = 72 \Leftrightarrow 2x2 = 72 \Leftrightarrow x2 = 36 \Leftrightarrow x = 6.$

Hence, height of the parallelogram = $6\ cm.$

Problem 33. The length of one side of a rhombus is 6.5 cm and its altitude is 10 cm. If the length of one of its diagonals is 26 cm, find the length of the other diagonal.

Sol. Area of rhombus = $(6.5 \times 10)cm^2 = 65\ cm^2.$

Let the length of the other diagonal be x cm.

Then, $\frac{1}{2} \times 26 \times x = 65$ or $x = 5cm$

Hence, length of the other diagonal = $5\ cm.$

Problem 34. Find the length of a rope by which a cow must be tethered in order that it may be able to graze an area of 9856 sq. metres.

Sol. Clearly, the cow will graze a circular field of area 9856 sq. metres and radius equal to the length of the rope.

Let the length of the rope be R metres.

Then, $\pi R^2 = 9856 \Leftrightarrow R^2 = \left(9856 \times \frac{7}{22}\right) = 3136$

$\Leftrightarrow R = 56$

∴ Length of the rope = $56\ m.$

Problem 35. The ratio of the circumferences of two circles is 2 : 3. What is the ratio of their areas?

Sol. Let the radius of the circles be *r* and *R* respectively.

Then, $\frac{2\pi r}{2\pi R} = \frac{2}{3} \Rightarrow \frac{r}{R} = \frac{2}{3}$

$\Rightarrow \frac{r^2}{R^2} = \left(\frac{2}{3}\right)^2$

$\Rightarrow \frac{\pi r^2}{\pi R^2} = \frac{4}{9}$

Hence, ratio of areas = $4 : 9.$

Problem 36. A circular wire of diameter 42 cm is bent in the form of a rectangle whose sides are in the ratio 6 : 5. Find the area of the rectangle.

Sol. We have: $r = 21\ cm.$

Perimeter of the rectangle = Circumference of the circle

$$= \left(2 \times \frac{22}{7} \times 21\right) cm = 132cm$$

Let the sides of the rectangle be $6x$ and $5x$.

Then, $2\,(6x + 5x) = 132 \Rightarrow 11x = 66 \Rightarrow x = 6.$

So, the sides of the rectangle are 36 cm and 30 cm.

∴Area of the rectangle = $(36 \times 30)cm^2 = 1080\ cm^2$.

Problem 37. The diameter of the driving wheel of a bus is 140 cm. How many revolutions per minute must the wheel make in order to keep a speed of 66 kmph?

Sol. Distance to be covered in 1 min= $\left(\frac{66\times100}{60}\right)m = 1100m$

Circumference of the wheel = $\left(2 \times \frac{22}{7} \times 0.70\right)m = 4.4m$

∴ Number of revolutions per min = $\left(\frac{1100}{4.4}\right) = 250$

Problem 38. A wheel makes 1000 revolutions in covering a distance of 88 km. Find the radius of the wheel.

Sol. Distance covered in one revolution = $\left(\frac{88\times1000}{1000}\right)m = 88m$

∴ $2\pi R = 88 \Leftrightarrow 2 \times \frac{22}{7} \times R = 88$

$\Leftrightarrow R = \left(88 \times \frac{7}{44}\right) = 14m.$

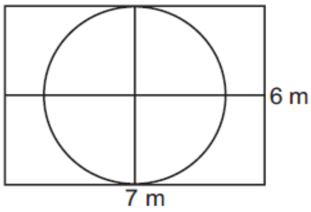
Problem 39. In a circle of radius 28 cm, an arc subtends an angle of 72° at the centre. Find the length of the arc and the area of the sector so formed.

Sol. $r = 28\ cm, \theta = 72°$.

∴Length of arc = $\left(2 \times \frac{22}{7} \times 28 \times \frac{72}{360}\right) cm = 35.2cm$

Area of the sector = $\left(\frac{22}{7} \times 28 \times 28 \times \frac{72}{360}\right) cm^2 = 492.8\ cm^2$

Problem 40. Find the area of the largest circle that can be drawn inside a rectangle with sides 7 m by 6 m.

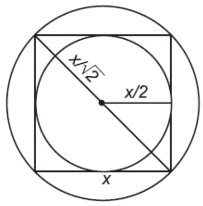


Sol. Radius of the required circle = $\left(\frac{1}{2} \times 6\right) m = 3m$

∴Area of the circle = $\left(\frac{22}{7} \times 3 \times 3\right) m^2$

$$= \left(\frac{198}{7}\right) m^2 = 28\frac{2}{7} m^2$$

Problem 41. Find the ratio of the areas of the incircle and circumcircle of a square.

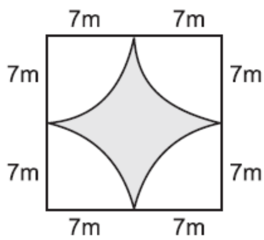


Sol. Let the side of the square be x. Then, its diagonal = $\sqrt{2}\, x$.

Radius of incircle = $\frac{x}{2}$ and radius of circumcircle = $\frac{\sqrt{2}x}{2} = \frac{x}{\sqrt{2}}$

∴ Required ratio = $\left(\frac{\pi x^2}{4} : \frac{\pi x^2}{2}\right) = \frac{1}{4} : \frac{1}{2} = 1:2$

Problem 42. Four horses are tied on the four corners of a square field of length 14 m so that each horse can just touch the other two horses. They were able to graze in the area accessible to them for 11 days. For how many days is the ungrazed area sufficient for them?



Sol. Area of the square field = $(14 \times 14) m^2 = 196\, m^2$.

Area accessible to the horses for grazing

$= 4 \times$ Area of a quadrant with $r = 7\, m$

= Area of a circle with $r = 7\, m = \left(\frac{22}{7} \times 7 \times 7\right) m^2 = 154 m^2$

Ungrazed area = $(196 - 154) m^2 = 42\, m^2$.

$154\, m^2$ area feeds the horses for 11 days.

∴ $42\, m^2$ area will feed the horses for $\left(\frac{11}{154} \times 42\right)\, days = 3 days$.

Problem 43. If the radius of a circle is decreased by 50%, find the percentage decrease in its area.

Sol. Let original radius $= R$. New radius $= \frac{50}{100}R = \frac{R}{2}$

Original area $= \pi R^2$ and New area $= \pi\left(\frac{R}{2}\right)^2 = \frac{\pi R^2}{4}$

$\therefore$ Decrease in area $= \left(\frac{3\pi R^2}{4} \times \frac{1}{\pi R^2} \times 100\right)\% = 75\%.$

Problem 44. If the radius of a circle is increased by 20% then by how much will its area be increased?

Sol. Let original radius $= R$. New radius $= \frac{120}{100}R = \frac{6R}{5}$

Original area $= \pi R^2$ and New area $= \pi\left(\frac{6R}{5}\right)^2 = \frac{36\pi R^2}{25}$

Increase in area $= \left(\frac{36\pi R^2}{25} - \pi R^2\right) = \frac{11\pi R^2}{25}$

$\therefore$ Increase % $= \left(\frac{11\pi R^2}{25} \times \frac{1}{\pi R^2} \times 100\right)\% = 44\%.$

Problem 45. The radius of a cricle is so increased that its circumference increased by 5%. Find the percentage increase in its area.

Sol. Let the original radius of the circle be r. Then, original circumference $= 2\pi r$.

New circumference $= 105\%$ of $(2\pi r) = \left(\frac{105}{100} \times 2\pi r\right) = 2\pi\left(\frac{21}{20}r\right)$

$\therefore$New radius $= \frac{21}{20}r$

New area $= \pi \times \left(\frac{21}{20}r\right)^2 = \frac{441}{400}\pi r^2$

Increase in area $= \left(\frac{441}{400}\pi r^2 - \pi r^2\right) = \frac{41}{400}\pi r^2$

Increase % in area $= \left(\frac{41}{400}\pi r^2 \times \frac{1}{\pi r^2} \times 100\right)\% = 10.25\%.$

Exercise

1. The circumference of a circle is equal to the side of a square whose area measures 407044 sq. cm. What is the area of the circle?
2. If the ratio between the areas of two circles is 4 : 1 then the ratio between their radii will be
3. The areas of two circular fields are in the ratio 16 : 49. If the radius of the latter is 14 m, then what is the radius of the former?
4. The radius of the wheel of a vehicle is 70 cm. The wheel makes 10 revolutions in 5 seconds. The speed of the vehicle is
5. Wheels of diameters 7 cm and 14 cm start rolling simultaneously from X and Y, which are 1980 cm apart, towards each other in opposite directions. Both of them make the same number of revolutions per second. If both of them meet after 10 seconds, the speed of the smaller wheel is
6. A small ring of negligible thickness and radius 2 cm moves on a bigger rung of radius 10 cm. How many rotations will the small ring take on the bigger ring to make a complete round?
7. A circular grassy plot of land, 42 cm in diameter, has a path 3.5 m wide running around it outside. The cost of gravelling the path at Rs. 4 per square metre is
8. The ratio of the outer and the inner perimeters of a circular path is 23 : 22. If the path is 5 metres wide, the diameter of the inner circle is
9. Total area of 64 small squares of a chessboard is 400 sq. cm. There is 3 cm wide border around the chess board. What is the length of the side of the chessboard?
10. A room is $12\frac{1}{4}$ m long and 7 m wide. The maximum length of a square tile to fill the floor of the room with whole number of tiles should be
11. Perimeter of a rectangular field is 160 metres and the difference between its two adjacent sides is 48 metres. The side of a square field, having the same area as that of the rectangle, is
12. Of the two square fields, the area of one is 1 hectare while the other one is broader by 1%. The difference in their areas is
13. The ratio of the area of a square to that of the square drawn on its diagonal, is
14. What will be the area of 4 metre high wall on all four sides of a rectangular hall having perimeter 64 m?

15. In a triangle ABC, a line XY is drawn parallel to BC meeting AB in X and AC in Y. The area of the triangle AXY is half of the area of the triangle ABC. XY divides AB in the ratio of
16. In an isosceles triangle, the measure of each of the equal sides is 10 cm and the angle between them is 45º. The area of the triangle is
17. What will be the ratio between the area of a rectangle and the area of a triangle with one of the sides of the rectangle as base and a vertex on the opposite side of the rectangle ?
18. Four equilateral triangles are described on the four sides of a rectangle with perimeter 12 cm. If the sum of the areas of the four triangles is $10\sqrt{3}\ cm^2$, what is the area of the rectangle?
19. The perimeter of a rhombus is 56 m and its height is 5 m. Its area is
20. The diameter of a circle is 3.5 cm. What is the circumference of the circle?
21. The distance between the parallel sides of a trapezium = The distance between the mid-points of the slant sides = 4 cm. What is the area of the trapezium?
22. A circle and a rectangle have the same perimeter. The sides of the rectangle are 18 cm and 26 cm. What is the area of the circle?
23. The perimeter of a square is equal to twice the perimeter of a rectangle of length 8 cm and breadth 7 cm. What is the circumference of a semi-circle whose diameter is equal to the side of the square? (rounded off to two decimal places)
24. An athletic track 14 m wide consists of two straight sections 120 m long joining semi-circular ends whose inner radius is 35 m. The area of the track is
25. If the circumference of a circle is 100 units, then what will be the length of the arc described by an angle of 20 degrees?
26. The area of a circle inscribed in an equilateral triangle is $154\ cm^2$. Find the perimeter of the triangle.
27. The diameter of a circle is equal to the perimeter of a square whose area is $3136\ cm^2$. What is the circumference of the circle?
28. A courtyard is 25m long and 16m broad is to be paved with bricks of dimensions 20cm by 10cm. What is the total number of bricks required?
29. A skating champion moves along the circumference of a circle of radius 28 m in 44 sec. How many seconds will it take her to move along the perimeter of a hexagon of side 48 m?

30. A one-rupee coin is placed on a plain paper. How many coins of the same size can be placed round it so that each one touches the centre and adjacent coins?

Answers

1. $32378.5\ cm^2$. 2. $\frac{2}{1}$ 3. $8m$ 4. 31.68 km/hr 5. 22 cm/s

6. 5 7. Rs. 2002 8. 220 m 9. 26 cm 10. 175 cm

11. 32 m 12. $201\ m^2$ 13. 1 : 2 14. $256\ m^2$ 15. $\frac{1}{(\sqrt{2}-1)}$

16. $25\sqrt{2}\ cm^2$ 17. 2 : 1 18. $8\ cm^2$ 19. $70\ m^2$ 20. 11 cm

21. $16\ cm^2$ 22. $616\ cm^2$ 23. 23.57 cm 24. $7056\ m^2$ 25. 5.55 units

26. 72.7 cm (approx.) 27. $704\ cm$ 28. 20000 bricks 29. 72 sec 30. 6

UNIT – 5

VOLUME AND SURFACE AREA

IMPORTANT FORMULAE:

I. Cuboid

Let length = l, breadth = b and height = h units. Then,

1) Volume $= (l \times b \times h)$ cubic units.

2) Surface area $= 2(lb + bh + lh)$ sq. units.

3) Diagonal $=\sqrt{l^2 + b^2 + h^2}$ units.

II. Cube

Let each edge of a cube be of length a. Then,

1) Volume $= a^3$ cubic units.

2) Surface area $= 6a^2$ sq. units.

3) Diagonal = 3 a units.

III. Cylinder

Let radius of base = r and Height (or length) = h. Then,

1) Volume $= (\pi r^2 h)$ cubic units.

2) Curved surface area $= (2\pi rh)$ sq. units.

3) Total surface area $= (2\pi rh + 2\pi r^2)$ sq. units

$= 2\pi r(h + r)$ sq. units

IV. Cone

Let radius of base = r and Height = h. Then,

1) Slant height, $l = h^2 + r^2$ units.

2) Volume $= \left(\frac{1}{3}\pi r^2 h\right)$ cubic units.

3) Curved surface area $= (\pi rl)$ sq. units.

4) **Total surface area** $= (\pi rl + \pi r^2)$ sq. units.

V. Frustum of a Cone

When a cone is cut by a plane parallel to the base of the cone then the portion between the plane and the base is called the frustum of the cone. Let radius of base = *R*, radius of top = *r*, and height = h. Then,

1) Volume $= \frac{\pi h}{3}(R^2 + r^2 + Rr)$ cubic units.

2) Slant height, $\boldsymbol{l} = \sqrt{(R - r)^2 + h^2}$ units.

3) Lateral (or curved) surface area $= \pi . l\,(R + r)$ sq. units.

4) Total surface area $= \pi\,[R^2 + r^2 + l\,(R + r)]$ sq. units

VI. Sphere

Let the radius of the sphere be *r*. Then,

1) Volume $= \left(\frac{4}{3}\pi r^3\right)$ cubic units.

2) Surface area $= (4\pi r^2)$ sq. units.

VII. Hemisphere

Let the radius of a hemisphere be *r*. Then,

1) Volume $= \left(\frac{2}{3}\pi r^3\right)$ cubic units.

2) Curved surface area $= (2\pi r^2)$ sq. units.

3) Total surface area $= (3\pi r^2)$ sq. units.

VIII. Pyramid

1) Volume $= \frac{1}{3} \times area\ of\ base \times height.$

2) Whole surface area = Area of base + Area of each of the lateral faces

Remember: 1 litre = 1000 cm^3.

SOLVED PROBLEMS.

Problem 1.

Find the volume and surface area of a cuboid 16 m long, 14 m broad and 7 m high.

Sol.

Volume = $(16 \times 14 \times 7)m^3 = 1568\, m^3$.

Surface area = $[2\,(16 \times 14 + 14 \times 7 + 16 \times 7)]cm^2 = (2 \times 434)cm^2 = 868\, cm^2$.

Problem 2.

A room is 12 metres long, 9 metres broad and 8 metres high. Find the length of the longest bamboo pole that can be placed in it.

Sol.

Length of the longest pole = Length of the diagonal of the room

$= (12)^2 + 9^2 + 8^2\, m = 289\, m = 17m.$

Problem 3.

The volume of a wall, 5 times as high as it is broad and 8 times as long as it is high, is 12.8 cu. metres. Find the breadth of the wall.

Sol.

Let the breadth of the wall be *x* metres.

Then, Height = 5*x* metres and Length = 40*x* metres.

$\therefore x \times 5x \times 40x = 12.8 \Leftrightarrow x^3 = \frac{12.8}{200} = \frac{128}{2000} = \frac{64}{1000}$

So, $x = \frac{4}{10} m = \left(\frac{4}{100} \times 100\right) cm = 40cm$

Problem 4.

Find the number of bricks, each measuring 24 cm × 12 cm × 8 cm, required to construct a wall 24 m long, 8m high and 60 cm thick, if 10% of the wall is filled with mortar?

Sol.

Volume of the wall = (2400 × 800 × 60) cu. cm.

Volume of bricks = 90% of the volume of the wall.

$$= \left(\frac{90}{100} \times 2400 \times 800 \times 60\right) cu.cm$$

Volume of 1 brick = (24 × 12 × 8) cu. cm.

∴ Number of bricks = $\left(\frac{90}{100} \times \frac{2400 \times 800 \times 60}{24 \times 12 \times 8}\right) = 45000.$

Problem 5.

A rectangular sheet of paper, 10 cm long and 8 cm wide has squares of side 2 cm cut from each of its corners. The sheet is then folded to form a tray of depth 2 cm. Find the volume of this tray.

Sol.

Clearly, we have:

Length of the tray = (10 – 2 × 2) cm = 6 cm.

Breadth of the tray = (8 – 2 × 2) cm = 4 cm.

Depth of the tray = 2 cm.

Volume of the tray = $(6 \times 4 \times 2) cm^3 = 48\ cm^3$.

Problem 6. *The dimensions of an open box are 50 cm, 40 cm and 23 cm. Its thickness is 3 cm. If 1 cubic cm of metal used in the box weighs 0.5 gms, find the weight of the box.*

Sol.

Volume of the metal used in the box = External volume – Internal volume

$$= [(50 \times 40 \times 23) - (44 \times 34 \times 20)] cm^3$$

$$= 16080\ cm^3$$

∴ Weight of the metal = $\left(\frac{16080 \times 0.5}{1000}\right)$ kg = 8.04 kg.

Problem 7 *The surface area of a cube is 1734 sq. cm. Find its volume***.**

Sol.

Let the edge of the cube be *a*. Then,

$6a^2 = 1734 \Rightarrow a^2 = 289 \Rightarrow a = 17\ cm.$

$\therefore Volume = a^3 = (17)^3\ cm^3 = 4913\ cm^3.$

Problem 8. *Three solid cubes of sides 1 cm, 6 cm and 8 cm are melted to form a new cube. Find the surface area of the cube so formed.*

Sol.

Volume of new cube = $(1^3 + 6^3 + 8^3)cm^3 = 729\ cm^3$.

Edge of new cube = $\sqrt[3]{729}\ cm = 9\ cm$.

∴ Surface area of the new cube = $(6 \times 9 \times 9)cm^2 = 486\ cm^2$.

Problem 9. *Two cubes have their volumes in the ratio 1 : 27. Find the ratio of their surface areas.*

Sol.

Let their edges be *a* and *b*. Then,

$$\frac{a^3}{b^3} = \frac{1}{27}$$

$$\left(\frac{a}{b}\right)^3 = \left(\frac{1}{3}\right)^3$$

$$\left(\frac{a}{b}\right) = \frac{1}{3}$$

∴ Ratio of their surface areas = $\frac{6a^2}{6b^2} = \frac{a^2}{b^2} = \left(\frac{a}{b}\right)^2 = \frac{1}{9}$

i.e., 1 : 9.

Problem 10. *Find the volume, curved surface area and the total surface area of a cylinder with diameter of base 7 cm and height 40 cm.*

Sol.

Volume = $\pi r^2 h$

$= \left(\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 40\right)\ cm^3$

$= 1540\ \text{cm}^3$

Curved surface area= $2\pi rh = \left(2 \times \frac{22}{7} \times \frac{7}{2} \times 40\right)\ cm^3 = 880\ cm^3$

Total surface area $= 2\pi rh + 2\pi r^2$

$= 2\pi r(h + r)$

$= \left(2 \times \frac{22}{7} \times \frac{7}{2} \times (40 + 3.5)\right) cm^2$

$= 957\ cm^2$

Problem 11. *A well with 14 m inside diameter is dug 10 m deep. Earth taken out of it has been evenly spread all around it to a width of 21 m to form an embankment. Find the height of the embankment.*

Sol.

Volume of earth dug out $= \left(\frac{22}{7} \times 7 \times 7 \times 10\right) m^3 = 1540\ m^3$

Area of embankment $= \frac{22}{7} \times [(28)^2 - (7)^2] = \left(\frac{22}{7} \times 35 \times 21\right)\ m^2 = 2310\ m^2$

Height of embankment $= \left(\frac{\text{Volume}}{\text{Area}}\right) = \left(\frac{1540}{2310}\right) m = \frac{2}{3} m$

Problem 12. *Find the slant height, volume, curved surface area and the whole surface area of a cone of radius 21 cm and height 28 cm.*

Sol.

Here, r = 21 cm and h = 28 cm.

∴ Slant height, $l\ = \sqrt{r^2 + h^2} = \sqrt{(21)^2 + (28)^2} = \sqrt{1225} = 35\ cm$

Volume $= \frac{1}{3}\pi r^2 h = \left(\frac{1}{3} \times \frac{22}{7} \times 21 \times 21 \times 28\right) = 12936\ cm^3$

Curved surface area $= \pi rl = \left(\frac{22}{7} \times 21 \times 28\right) \text{cm}^2 = 2310\ \text{cm}^2$

Total surface area $= (\pi rl + \pi r^2) = \left(2310 + \frac{22}{7} \times 21 \times 21\right)\ cm^2 = 3696\ \text{cm}^2$.

Problem 13. *The heights of two right circular cones are in the ratio 1 : 2 and the perimeters of their bases are in the ratio 3 : 4. Find the ratio of their volumes.*

Sol.

Let the radii of their bases be *r* and *R* and their heights be *h* and 2*h* respectively.

Then, $\frac{2\pi r}{2\pi R} = \frac{3}{4} \Rightarrow \frac{\text{r}}{R} = \frac{3}{4}$

$\Rightarrow \text{R} = \frac{3}{4} r$

$\therefore$ Ratio of volumes $= \frac{\frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi\left(\frac{4}{3}r\right)^2(2h)} = \frac{9}{32} = 9:32.$

Problem 14. *A conical vessel, whose internal radius is 12 cm and height 50 cm, is full of liquid. The contents are emptied into a cylindrical vessel with internal radius 10 cm. Find the height to which the liquid rises in the cylindrical vessel.*

Sol.

Volume of the liquid in the cylindrical vessel

$= Volume\ of\ the\ conical\ vessel$

$= \left(\frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times 50\right) cm^3$

$= \left(\frac{22\times 4\times 12\times 50}{7}\right) cm^3$

Let the height of the liquid in the vessel be h.

Then, $\frac{22}{7} \times 10 \times 10 \times h = \frac{22\times 4\times 12\times 50}{7}$

$h = \left(\frac{4\times 12 \times 50}{7}\right) = 24\ cm$

Problem 15. *In a rocket shaped firecracker, explosive powder is to be filled up inside the metallic enclosure. The metallic enclosure is made up of a cylindrical base and conical top with the base of radius 8 cm. The ratio of height of cylinder and cone is 5 : 3. A cylindrical hole is drilled through the metal solid with height one-third the height of metal solid. What should be the radius of the hole, so that volume of the hole (in which gun powder is to be filled up) is half of the volume of metal after drilling?*

Sol.

Let the height of cylinder and cone be $5x$ and $3x$ cm respectively.

Then, height of metal solid = $(5x + 3x)$ cm = $8x$ cm.

Height of hole $= \left(\frac{8x}{3}\right)\ cm$

Radius of cylinder = Radius of cone = 8 cm.

Let the radius of the hole be r cm.

Volume of metal solid after drilling

$= \text{Volume of cylinder} + \text{Volume of cone} - \text{Volume of hole}$

$= \left(\pi \times 8^2 \times 5x \times \frac{1}{3}\pi \times 8^2 \times 3x \times \frac{8x}{3}\right) cm^3$

$= \left(320\, \pi x + 64\, \pi x - \pi r^2 . \frac{8x}{3}\right) cm^3$

$= \left(384\, \pi x - \pi r^2 . \frac{8x}{3}\right) cm^3$

$384\, \pi x - \pi r^2 . \frac{8x}{3} = 2\pi r^2 . \frac{8x}{3}$

$\Rightarrow 3\pi r^2 . \frac{8x}{3} = 384\pi x$

$\Rightarrow r^2 = \frac{384}{8} = 48$

$\Rightarrow r = 4\sqrt{3}$cm.

Problem 16. *How many spherical bullets can be made out of a lead cylinder 28 cm high and with base radius 6 cm, each bullet being 1.5 cm in diameter?*

Sol.

Volume of cylinder = $(\pi \times 6 \times 6 \times 28) cm^3$

$= (36 \times 28)\, \pi\, cm^3$.

Volume of each bullet = $\left(\frac{4}{3}\pi \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}\right) cm^3 = \frac{9\pi}{16}\, cm^3$

Number of bullets= $\frac{\text{Volume of cylinder}}{\text{Volume of each bullet}} = \left[(36 \times 28)\pi \times \frac{16}{9\pi}\right] = 1792.$

Problem 17. A hemispherical bowl of internal radius 9 cm contains a liquid. This liquid is to be filled into cylindrical shaped small bottles of diameter 3 cm and height 4 cm. How many bottles will be needed to empty the bowl?

Sol.

Volume of bowl = $\left(\frac{2}{3}\pi \times 9 \times 9 \times 9\right)\, cm^3 = 486\pi\, cm^3$

Volume of 1 bottle = $\left(\pi \times \frac{3}{2} \times \frac{3}{2} \times 4\right) = 9\pi\, cm^3$

Number of bottles = $\left(\frac{486\pi}{9\pi}\right) = 54.$

Problem 18. *A cylindrical container of radius 6 cm and height 15 cm is filled with ice-cream. The whole ice-cream has to be distributed to 10 children in equal cones with hemispherical tops. If the height of the conical portion is four times the radius of its base, then find the radius of the ice-cream cone.*

Sol.

Volume of ice-cream in cylindrical container = $(\pi \times 6^2 \times 15)cm^3 = (540\pi)cm^3$.

Let the radius of the base of the cone be *r* cm.

Then, height of the cone = (4*r*) cm.

Volume of ice-cream in 10 cones with hemispherical tops = $\left[10\left\{\frac{1}{3}\pi r^2 \times 4r + \frac{2}{3}\pi r^3\right\}\right] = (20\pi r^3)cm^3$

$$20\ \text{p}r^3 = 540\text{p}$$

$$\Rightarrow r^3 = \frac{540\pi}{20\pi} = 27$$

$$\Rightarrow r = 3$$

Hence, radius of ice-cream cone = 3 cm.

Exercise

1. A rectangular water tank is 8 m high, 6 m long and 2.5 m wide. How many litres of water can it hold?
2. A closed aquarium of dimensions 30 cm × 25 cm × 20 cm is made up entirely of glass plates held together with tapes. The total length of tape required to hold the plates together (ignore the overlapping tapes) is
3. The breadth of a room is twice its height and half its length. The volume of the room is 512 cu. m. The length of the room is
4. The length of the longest rod that can be placed in a room of dimensions 10 m × 10 m × 5 m is

5. A swimming bath is 24 m long and 15 m broad. When a number of men dive into the bath, the height of the water rises by 1 cm. If the average amount of water displaced by one of the men be 0.1 cu. m, how many men are there in the bath?
6. A rectangular tank measuring 5 m × 4.5 m × 2.1 m is dug in the centre of the field measuring 13.5 m by 2.5 m. The earth dug out is evenly spread over the remaining portion of the field. How much is the level of the field raised?
7. An open box is made by cutting the congruent squares from the corners of a rectangular sheet of cardboard of dimensions 20 cm × 15 cm. If the side of each square is 2 cm, the total outer surface area of the box is
8. A covered wooden box has the inner measures as 115 cm, 75 cm and 35 cm and the thickness of wood is 2.5 cm. Find the volume of the wood.
9. If a metallic cuboid weighs 16 kg, how much would a miniature cuboid of metal weigh, if all dimensions are reduced to one-fourth of the original?
10. If the areas of three adjacent faces of a cuboid are x, y, z respectively, then the volume of the cuboid is
11. The dimensions of a certain machine are 48″ × 30″× 52″. If the size of the machine is increased proportionately until the sum of its dimensions equals 156″, what will be the increase in the shortest side?
12. Each side of a cube measures 8 metres. What is the volume of the cube?
13. The surface area of a cube is $150\ cm^2$. Its volume is
14. If the total length of diagonals of a cube is 12 cm, then what is the total length of the edges of the cube?
15. How many cubes of 10 cm edge can be put in a cubical box of 1 m edge?
16. From a cube of side 8m, a square hole of 3m side is hollowed from end to end. What is the volume of the remaining solid?
17. How many small cubes, each of 96 cm surface area, can be formed from the material obtained by melting a larger cube of 384 cm surface area?
18. The volume of a cuboid is twice that of a cube. If the dimensions of the cuboid are 9 cm, 8 cm and 6 cm, the total surface area of the cube is
19. By what percent the volume of a cube increases if the length of each edge was increased by 50%?
20. A circular well with a diameter of 2 metres, is dug to a depth of 14 metres. What is the volume of the earth dug out?

21. Find the cost of a cylinder of radius 14 m and height 3.5 m when the cost of its metal is Rs.50 per cubic metre.
22. Capacity of a cylindrical vessel is 25.872 litres. If the height of the cylinder is three times the radius of its base, what is the area of the base?
23. Two cans have the same height equal to 21 cm. One can is cylindrical, the diameter of whose base is 10 cm. The other can has square base of side 10 cm. What is the difference in their capacities?
24. If the radius of the base of a right circular cylinder is halved, keeping the height same, what is the ratio of the volume of the reduced cylinder to that of the original one?
25. Diameter of a jar cylindrical in shape is increased by 25%. By what percent must the height be decreased so that there is no change in its volume?
26. Water flows out through a circular pipe whose internal diameter is 2 cm, at the rate of 6 metres per second into a cylindrical tank, the radius of whose base is 60 cm. By how much will the level of water rise in 30 minutes?
27. Find the number of coins 1.5 cm in diameter and 0.2 cm thick, to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.
28. What part of a ditch, 48 metres long, 16.5 metres broad and 4 metres deep can be filled by the earth got by digging a cylindrical tunnel of diameter 4 metres and length 56 metres ?
29. What is the weight of water contained in a conical vessel 21 cm deep and 16 cm in diameter?
30. If the heights of two cones are in the ratio 7 : 3 and their diameters are in the ratio 6 : 7, what is the ratio of their volumes?
31. A solid metallic right circular cylinder of base diameter 16 cm and height 2 cm is melted and recast into a right circular cone of height three times that of the cylinder. Find the curved surface area of the cone. [Use $\pi = 3.14$]
32. For a sphere of radius 10 cm, what percent of the numerical value of its volume would be the numerical value of the surface area?
33. If the radii of two spheres are in the ratio 1 : 4, then their surface areas are in the ratio
34. If the radius of a sphere is doubled, how many times does its volume become?
35. If three metallic spheres of radii 6 cms, 8 cms and 10 cms are melted to form a single sphere, the diameter of the new sphere will be
36. How many lead shots each 3 mm in diameter can be made from a cuboid of dimensions 9 cm × 11 cm × 12 cm?

37. A right circular cylinder and a sphere are of equal volumes and their radii are also equal. If h is the height of the cylinder and d, the diameter of the sphere, then
38. A copper wire of length 36 m and diameter 2 mm is melted to form a sphere. The radius of the sphere (in cm) is
39. A metallic sphere of radius 5 cm is melted to make a cone with base of the same radius. What is the height of the cone?
40. A hemispherical bowl is 176 cm round the brim. Supposing it to be half full, how many persons may be served from it in hemispherical glasses 4 cm in diameter at the top?

Answers

1. 120000 litres	2. 300cm	3. 16m	4. 15	5. 36
6. 4.2m	7. 284 cm^2	8. 82125 cu.cm	9. 0.25kg	
10. $\sqrt{xyz}$	11. $6''$	12. 512 cu.cm	13. $125cm^3$	
14. $15cm$	15. 1000	16. $440m^3$	17. 8	
19. 237.5%	20. $44m^3$	21. Rs. 107800	22. $616cm^2$	
23. 450 cm^3	24. 1:4	25. 36	26. $3m$	
27. 450	28. $\frac{2}{9}$	29. 1.408kg	30. 12:7	
31. $251.2cm^2$	32. 30%	33. 1:16	34. 8 times	
35. 24cms	36. 84000	37. $\frac{h}{2}=\frac{d}{3}$	38. 3	
39. 20 cm	40. 1372			

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